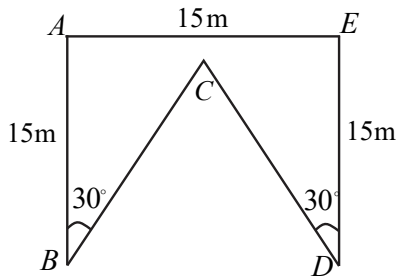


CSIR NET QUESTION PAPER

07-June-2023

PART - A:

Qus 1. The figure shows map of a field bounded by ABCDE. If AB and DE are perpendicular to AE, then the perimeter of the field is



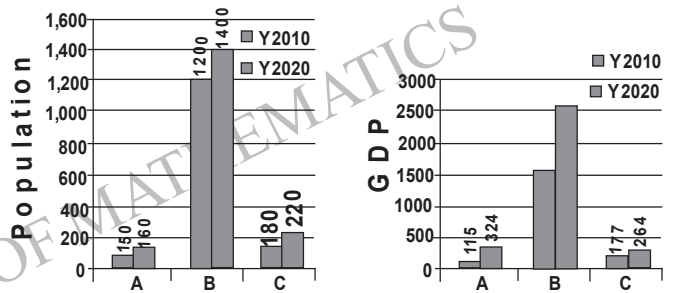
- (a) 70 m (b) 75 m
(c) 80 m (d) 85 m

Qus 2. An appropriate diagram to depict the relationships between the categories INSECTS, BIRDS, EXTINCT ANIMALS and PEACOCKS is

- (A)
- (B)
- (C)
- (D)

- (a) A (b) B
(c) C (d) D

Qus. 3 The populations and gross domestic products (GDP) in billion USD of three countries A, B and C in the year 2010 and 2020 are shown in the two figure below.



- (a) A,B,C (b) B,A,C
(c) B,C,A (d) C,A,B

Qus. 4 A device needs 4 batteries to run. Each battery runs for 2 days. If there are a total of 6 batteries available, what is the maximum number of days for which the device can be run by strategically replacing the batteries till all the batteries are completely drained of power?

- (a) 2 (b) 3
(c) 4 (d) 5

Qus. 5 A person takes loan of Rs. 1,50,000 at a compound interest rate of 10% per annum. If the loan is repaid at the end of the 3rd year, what is the total interest paid?

- (a) 45000 (b) 82600
(c) 94600 (d) 49650

Qus. 6 In a group of 7 people, 4 have exactly one sibling and 3 have exactly two siblings. Two people selected at random from the group, what is the probability that they are NOT siblings?

- (a) 5/21 (b) 16/21
(c) 3/7 (d) 4/7

Qus. 7 Consider the following paragraph:

THE ABILITY OF REASON ACCURATELY IS VERY IMPORTANT, AS IS THE ABILITY TO COUNT, AS AN EXERCISE BOTH, LET US COUNT HOW MANY TIMES THE LETTER "E" OCCURS IN THIS PARAGRAPH, THE CORRECT COUNT IS _____

Which option when put in the blank in the above paragraph will make the final sentence accurate?

- | | |
|--------------|---------------|
| (a) SIXTEEN | (b) SEVENTEEN |
| (c) EIGHTEEN | (d) NINETEEN |

Qus. 8 The difference of the squares of two distinct two-digit numbers with one being obtained by reversing the digits of the other is always divisible by

- | | |
|--------|--------|
| (a) 4 | (b) 6 |
| (c) 10 | (d) 11 |

Qus. 9 Two datasets A and B have the same mean. Which of the following MUST be true?

- (a) Sum of the observations in A = Sum of the observations in B
- (b) Mean of the squares of the observations in A = Mean of the squares of the observations in B
- (c) If the two datasets are combined, then the mean of the combined dataset = mean of A + mean of B
- (d) If the two datasets are combined, then the mean of the combined dataset = mean of A

Qus. 10 A and B have in their collection, coins of Rs. 1, Rs. 2, Rs. 5 and Rs. 10 in the ratio 3:2:2:1 and 4:3:2:1, respectively. The total number of coins with each of them is equal. If the value of coins with A is Rs. 270/-, what is the value of the coins (in Rs) with B?

- | | |
|---------|---------|
| (a) 213 | (b) 240 |
| (c) 275 | (d) 282 |

Qus. 11 If the speed of a train is increased by 20% its travel time between two stations reduces by 2 hrs. If its speed is decreased by 20%, the travel time increases by 3 hrs. What is the normal duration of travel (in hrs.)?

- | | |
|----------|----------|
| (a) 11.5 | (b) 12.0 |
| (c) 13.2 | (d) 14.0 |

Qus. 12 In an examination containing 10 questions, each correct answer is awarded 2 marks, each incorrect answer is awarded -1 and each unattempted question is awarded zero. Which of the following CANNOT be a possible score in the examination?

- | | |
|--------|--------|
| (a) -9 | (b) -7 |
|--------|--------|

- | | |
|--------|--------|
| (c) 17 | (d) 19 |
|--------|--------|

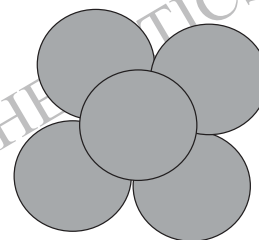
Qus. 13 Person A tells the truth 30% of the times and B tells the truth 40% of the times, independently. What is the minimum probability that they would contradict each other?

- | | |
|----------|----------|
| (a) 0.18 | (b) 0.42 |
| (c) 0.46 | (d) 0.50 |

Qus. 14 The ratio of ages of a mother and daughter is 14:1 at present. After four years, the ratio of their ages will be 16:3. What was the age of mother when the daughter was born?

- | | |
|--------|--------|
| (a) 26 | (b) 28 |
| (c) 30 | (d) 32 |

Qus. 15 Five identical incompressible sphere of radius 1 unit are stacked in a pyramidal form as shown in the figure. The height of the structure is



Top view

- | | |
|-----------------------|--------------------|
| (a) $2 + \sqrt{2}$ | (b) $2 + \sqrt{3}$ |
| (c) $2 + 2\sqrt{2/3}$ | (d) 3 |

Qus. 16 A boy can escape through a window of size at least 4 feet. The 28 windows of a house are of sizes 2, 3, 4 or 5 feet and their numbers are proportional to their sizes. The number of windows available for the boy to escape through is

- | | |
|--------|--------|
| (a) 2 | (b) 9 |
| (c) 10 | (d) 18 |

Qus. 17 The standard deviation of data $x_1, x_2, x_3, \dots, x_n$ is σ ($\sigma > 0$). Then the standard deviation of data $3x_1 + 2, 3x_2 + 2, 3x_3 + 2, \dots, 3x_n + 2$ is

- | | |
|-------------------|---------------|
| (a) 3σ | (b) σ |
| (c) $3\sigma + 2$ | (d) 9σ |

Qus. 18 On a spherical globe of radius 10 units, the distance between A and B is 25 units. If it is uniformly expanded to a globe of radius 50 units, the distance between them in the

same units would be

- | | |
|---------|---------|
| (a) 75 | (b) 125 |
| (c) 150 | (d) 625 |

Qus. 19 In a meeting of 45 people, there are 40 people who know one another and the remaining know no one. People who know each other only hug, whereas those who do not know each other only shake hands. How many handshakes occur in this meeting?

- | | |
|---------|---------|
| (a) 225 | (b) 10 |
| (c) 210 | (d) 200 |

Qus. 20 In an assembly election, parties A,B,C,D and E won 30, 25, 20, 10 and 4 seatsm, respectively, whereas independents won 9 sets. Based on this data, which of the following statements must be INCORRECT?

- (a) No party has majority
(b) A and C together can form the goverment
(c) A and D with the support of independents get the majority
(d) An MLA from E can become Chief Minister

PART - B:-

Qus. 21 Let $x = (x_1, \dots, x_n)$ and $y = (y_1, \dots, y_n)$ denote vectors in \mathbb{R}^n for a fixed $n \geq 2$. Which of the following defines an inner product on \mathbb{R}^n ?

- (a) $\langle x, y \rangle = \sum_{i,j=1}^n x_i y_j$
(b) $\langle x, y \rangle = \sum_{i,j=1}^n (x_i^2 + y_j^2)$
(c) $\langle x, y \rangle = \sum_{j=1}^n j^3 x_j y_j$
(d) $\langle x, y \rangle = \sum_{j=1}^n j^3 x_j y_{n-j+1}$

Qus. 22 Consider the constants a and b such that the following generalized coordinate transformation from (p, q) to (P, Q) is canonical

$$Q = pq^{(a+1)}, P = q^b.$$

What are the values of a and b ?

- (a) $a = -1, b = 0$
(b) $a = -1, b = 1$
(c) $a = 1, b = 0$
(d) $a = 1, b = -1$

Qus. 23 Consider the series $\sum_{n=1}^{\infty} a_n$, where

$a_n = (-1)^{n+1} (\sqrt{n+1} - \sqrt{n})$. Which of the following statements is true?

- (a) The series is divergent
(b) The series is convergent
(c) The series is conditionally convergent
(d) The series is absolutely convergent

Qus. 24 Let $l \geq 1$ be a positive integer. What is the dimension of the \mathbb{R} vector space of all polynomials in two variables over \mathbb{R} having a total degree of at most l ?

- | | |
|----------------|--------------------|
| (a) $l+1$ | (b) $l(l-1)$ |
| (c) $l(l+1)/2$ | (d) $(l+1)(l+2)/2$ |

Qus. 25 Consider the function f defined by

$$f(z) = \frac{1}{1-z-z^2} \quad \text{for } z \in \mathbb{C} \text{ such that}$$

$1-z-z^2 \neq 0$. Which of the following statements is true?

- (a) f is an entire function
(b) f has a simple pole at $z = 0$
(c) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficient a_n are recursively defined as follows $a_0 = 1, a_1 = 0$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$
(d) f has a Taylor series expansion $f(z) = \sum_{n=0}^{\infty} a_n z^n$, where coefficients a_n are recursively defined as follows: $a_0 = 1, a_1 = 1$ and $a_{n+2} = a_n + a_{n+1}$ for $n \geq 0$

Qus. 26 How many real roots does the polynomial $x^3 + 3x - 2023$ have?

- | | |
|-------|-------|
| (a) 0 | (b) 1 |
| (c) 2 | (d) 3 |

Qus. 27 Let $\{\epsilon_n : n \geq 1\}$ represent the results of independent rolls of a dice with probability of the face i turning up being $p_i > 0$ for

$i = 1, 2, \dots, 6$ and $\sum_{i=1}^6 p_i = 1$. Let $\{X_n : n \geq 0\}$ be the Markov chain on the state space $\{1, 2, \dots, 6\}$ where $X_n = \max\{\epsilon_1, \epsilon_2, \dots, \epsilon_{n+1}\}$.

Then $\lim_{n \rightarrow \infty} P(X_n = 4 | X_0 = 3)$ equals

- (a) p_4 (b) 1
(c) $1 - p_3$ (d) 0

Qus. 28 For the unknown $y : [0, 1] \rightarrow \mathbb{R}$, consider the following two-point boundary value problem

$$\text{lem } \begin{cases} y''(x) + 2y(x) = 0 & \text{for } x \in (0, 1) \\ y(0) = y(1) = 0 \end{cases}. \text{ It}$$

is given that the above boundary value problem corresponds to the following integral equation:

$$y(x) = 2 \int_0^1 K(x, y) y(t) dt \text{ for } x \in [0, 1]$$

Which of the following is the kernel $K(x, t)$?

- (a) $K(x, t) = \begin{cases} t(1-x) & \text{for } t < x \\ x(1-t) & \text{for } t > x \end{cases}$
(b) $K(x, t) = \begin{cases} t^2(1-x) & \text{for } t < x \\ x^2(1-t) & \text{for } t > x \end{cases}$
(c) $K(x, t) = \begin{cases} \sqrt{t}(1-x) & \text{for } t < x \\ \sqrt{x}(1-t) & \text{for } t > x \end{cases}$
(d) $K(x, t) = \begin{cases} \sqrt{t^3}(1-x) & \text{for } t < x \\ \sqrt{x^3}(1-t) & \text{for } t > x \end{cases}$

Qus. 29 Let A be a 3×3 real matrix whose characteristic polynomial $p(T)$ is divisible by T^2 .

Which of the following statements is true?

- (a) The eigenspace of A for the eigenvalue 0 is two-dimensional
(b) All the eigenvalues of A are real
(c) $A^3 = 0$
(d) A is diagonalizable

Qus. 30 Suppose X_1, X_2, \dots, X_n are independently and identically distributed $N(\theta, 1)$ random variables, for $\theta \in \mathbb{R}$. Suppose

$\bar{X} = n^{-1} \sum_{i=1}^n X_i$ denotes the sample mean

and let $t_{0.975, n-1}$ denote the 0.975 quantile of a student s-t distribution with $n-1$ degree of freedom. Which of the following statements is true for the following interval

$$\bar{X} \pm t_{0.975, n-1} \frac{1}{\sqrt{n}}?$$

- (a) The interval is a confidence interval for θ with confidence level of exactly 0.95
(b) The interval is a confidence interval for θ with confidence level being less than 0.95
(c) The interval is a confidence interval for θ with confidence level being more than 0.95
(d) The interval is not a confidence interval

Qus. 31 Let C be the positively oriented circle in the complex plane of radius 3 centered at the origin. What is the value of the integral

$$\oint_C \frac{dz}{z^2 (e^z - e^{-z})}?$$

- (a) $i\pi/12$ (b) $-i\pi/12$
(c) $i\pi/6$ (d) $-i\pi/6$

Qus. 32 Consider the simple linear regression model $Y_i = \beta x_i + \epsilon_i$, for $i = 1, \dots, n$; where $E(\epsilon_i) = 0$, $\text{cov}(\epsilon_i, \epsilon_k) = 0$ if $i \neq k$ and $\text{Var}(\epsilon_i) = x_i^2 \sigma^2$. The best linear unbiased estimator of β is:

- (a) $\frac{\sum_{i=1}^n Y_i x_i}{\sum_{i=1}^n x_i^2}$ (b) $\frac{\sum_{i=1}^n Y_i}{\sum_{i=1}^n x_i}$
(c) $\frac{1}{n} \sum_{i=1}^n \frac{Y_i}{x_i}$ (d) $\frac{1}{n} \sum_{i=1}^n \frac{Y_i x_i}{x_i^2}$

Qus. 33 Which of the following functions is uniformly continuous on the interval $(0, 1)$?

(a) $f(x) = \sin \frac{1}{x}$

(b) $f(x) = e^{-1/x^2}$

(c) $f(x) = e^x \cos \frac{1}{x}$

(d) $f(x) = \cos x \cos \frac{\pi}{x}$

Qus. 34 Let $u(x, t)$ be the solution of

$$u_{tt} - u_{xx} = 0, \quad 0 < x < 2, t > 0$$

$$u(0, t) = 0 = u(2, t), \quad \forall t > 0$$

$$u(x, 0) = \sin(\pi x) + 2 \sin(2\pi x), \quad 0 \leq x \leq 2$$

$$u_t(x, 0) = 0 \quad 0 \leq x \leq 2$$

Which of the following is true?

(a) $u(1, 1) = 1$

(b) $u(1/2, 1) = 0$

(c) $u(1/2, 2) = 1$

(d) $u_t(1/2, 1/2) = \pi$

Qus. 35 Which of the following is a valid cumulative distribution function?

(a) $F(x) = \begin{cases} 1 & \text{if } x < 0 \\ \frac{2+x^2}{3+x^2} & \text{if } x \geq 0 \end{cases}$

(b) $F(x) = \begin{cases} \frac{1}{2+x^2} & \text{if } x < 0 \\ \frac{2+x^2}{3+2x^2} & \text{if } x \geq 0 \end{cases}$

(c) $F(x) = \begin{cases} \frac{1}{2+x^2} & \text{if } x < 0 \\ \frac{2 \cos(x) + x^2}{4+x^2} & \text{if } x \geq 0 \end{cases}$

(d) $F(x) = \begin{cases} \frac{1}{2+x^2} & \text{if } x < 0 \\ \frac{1+x^2}{4+x^2} & \text{if } x \geq 0 \end{cases}$

Qus. 36 Let X_1, X_2, X_3 and X_4 be independent and identically distributed Bernoulli $\left(\frac{1}{3}\right)$ random

variables. Let $X_{(1)}, X_{(2)}, X_{(3)}$ and $X_{(4)}$ denote the corresponding order statistics. Which of the following is true?

(a) $X_{(1)}$ and $X_{(4)}$ are independent

(b) Expectation of $X_{(2)}$ is $\frac{1}{2}$

(c) Variance of $X_{(2)}$ is $\frac{8}{81}$

(d) $X_{(4)}$ is a degenerate random variable

Qus. 37 Let X be a Poisson random variable with mean λ . Which of the following parametric function is not estimable?

(a) λ^{-1}

(b) λ

(c) λ^2

(d) $e^{-\lambda}$

Qus. 38 Suppose $X = (X_1, X_2, X_3, X_4)^T$ has multi-

variate normal $N_4(0, I_2 \otimes \Sigma)$, where I_2 is the 2×2 identity matrix \otimes is the Kronecker

product, and $\Sigma = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Define

$$Z = \begin{pmatrix} X_1 & X_2 \\ X_3 & X_4 \end{pmatrix} \text{ and } Q = ((Q_{ij})) = Z^T Z. \text{ Suppose}$$

χ_n^2 denotes a chi-square random variate with n degrees of freedom, and $W_m(n, \Sigma)$ denotes a Wishart distribution of order m with parameters n and Σ . The distribution of $(Q_{11} + Q_{12} + Q_{21} + Q_{22})$ is

- (a) $W_1(2,2)$ (b) $W_1(1,2)$
(c) $W_1(2,1)$ (d) $2\chi_4^2$

Qus. 39 Let T be a linear operator on \mathbb{R}^3 . Let

$f(X) \in \mathbb{R}[X]$ denote its characteristic polynomial. Consider the following statements.

- (A) Suppose T is non-zero and 0 is an eigenvalue of T . If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator $g(T)$ is zero.
(B) Suppose 0 is an eigenvalue of T with at least two linearly independent eigen vectors. If we write $f(X) = Xg(X)$ in $\mathbb{R}[X]$, then the linear operator $g(T)$ is zero.

Which of the following is true?

- (a) Both (A) and (B) are true
(b) Both (A) and (B) are false
(c) (A) is true and (B) is false
(d) (A) is false and (B) is true

Qus. 40 Which of the following assertions is correct?

- (a) $\limsup_n e^{\cos\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} > 1$
(b) $\lim_n e^{\log e^{\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)}}$ does not exist
(c) $\liminf_n e^{\sin\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} < \pi$
(d) $\lim_n e^{\tan\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)}$ does not exist

Qus. 41 Let $x, y \in [0, 1]$ be such that $x \neq y$. Which of the following statements is true for every $\epsilon > 0$?

- (a) There exists a positive integer N such that $|x - y| < 2^{-n} \in$ for every integer $n \geq N$
(b) There exists a positive integer N such that $2^{-n} \in < |x - y|$ for every integer $n \geq N$

- (c) There exists a positive integer N such that $|x - y| < 2^{-n} \in$ for every integer $n \geq N$
(d) For every positive integer $N, |x - y| < 2^{-n} \in$ for some integer $n \geq N$

Qus. 42 Consider the quadratic form $Q(x, y, z)$

associated to the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$.

Let

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid Q(a, b, c) = 0 \right\}.$$

Which of the following statements is false?

- (a) The intersection of S with the xy -plane is a line
(b) The intersection of S with the xz -plane is an ellipse
(c) S is the union of two planes
(d) Q is a degenerate quadratic form

Qus. 43 Consider the random sample $\{3, 6, 9\}$ of size 3 from a normal distribution with mean $\mu \in (-\infty, 5]$ and variance 1. Then the maximum likelihood estimate of μ is

- (a) 6 (b) 5
(c) 3 (d) 9

Qus. 44 Consider the linear programming problem
maximize $x + 3y$

subject to $A \begin{pmatrix} x \\ y \end{pmatrix} \leq b$

where $A = \begin{pmatrix} -1 & -1 \\ 0 & 1 \\ -1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} -1 \\ 5 \\ 5 \\ 14 \\ 0 \end{pmatrix}$

Which of the following statements is true?

- (a) The objective function attains its maximum at $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ in the feasible region.
- (b) The objective function attains its maximum at $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ in the feasible region.
- (c) The objective function attains its maximum at $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in the feasible region.
- (d) The objective function does not attain its maximum at $\begin{pmatrix} 14 \\ 0 \end{pmatrix}$ in the feasible region.

Qus. 45 Consider the variational problem (P)

$$J(y(x)) = \int_0^1 [(y')^2 - y|y|y' + xy] dx,$$

$$y(0) = 0, y(1) = 0.$$

Which of the following statements is correct?

- (a) (P) has no stationary function (extremal)
- (b) $y \equiv 0$ is the only stationary function (extremal) for (P)
- (c) (P) has a unique stationary function (extremal) y not identically equal to 0
- (d) (P) has infinitely many stationary functions (extremal)

Qus. 46 Which of the following equations can occur as the class equation of a group of order 10?

- (a) $10 = 1 + 1 + \dots + 1$ (10 times)
- (b) $10 = 1 + 1 + 2 + 2 + 2 + 2$
- (c) $10 = 1 + 1 + 1 + 2 + 5$
- (d) $10 = 1 + 2 + 3 + 4$

Qus. 47 Let $u(x, y)$ be the solution of the Cauchy problem

$$u u_x + u_y = 0, \quad x \in \mathbb{R}, y > 0$$

$$u(x, 0) = x, \quad x \in \mathbb{R}$$

Which of the following is the value of $u(2, 3)$?

- (a) 2 (b) 3
- (c) 1/2 (d) 1/3

Qus. 48 Suppose $x(t)$ is the solution of the following initial value problem in \mathbb{R}^2

$$\dot{x} = Ax, \quad x(0) = x_0, \quad \text{where } A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

Which of the following statements is true?

- (a) $x(t)$ is a bounded solution for some $x_0 \neq 0$
- (b) $e^{-6t} |x(t)| \rightarrow 0$ as $t \rightarrow \infty$, for all $x_0 \neq 0$
- (c) $e^{-t} |x(t)| \rightarrow \infty$ as $t \rightarrow \infty$ for all $x_0 \neq 0$
- (d) $e^{-10t} |x(t)| \rightarrow 0$ as $t \rightarrow \infty$ for all $x_0 \neq 0$

Qus. 49 Which of the following values of a, b, c and d will produce a quadrature formula

$$\int_{-1}^1 f(x) dx \approx a f(-1) + b f(1) + c f'(-1) + d f'(1)$$

that has degree of precision 3?

- (a) $a = 1, b = 1, c = \frac{1}{3}, d = -\frac{1}{3}$
- (b) $a = -1, b = 1, c = \frac{1}{3}, d = -\frac{1}{3}$
- (c) $a = 1, b = 1, c = -\frac{1}{3}, d = \frac{1}{3}$
- (d) $a = 1, b = -1, c = \frac{1}{3}, d = -\frac{1}{3}$

Qus. 50 Suppose S is an infinite set. Assuming that the axiom of choice holds, which of the following is true?

- (a) S is in bijection with the set of rational numbers
- (b) S is in bijection with the set of real numbers
- (c) S is in bijection with $S \times S$
- (d) S is in bijection with the power set of S

Qus. 51 Let $X = (X_1, X_2)^T$ follow a bivariate normal distribution with mean vector $(0, 0)^T$ and covariance matrix Σ such that

$$\Sigma = \begin{bmatrix} 5 & -3 \\ -3 & 10 \end{bmatrix}.$$

The mean vector and

covariance matrix of $Y = (X_1, 5 - 2X_2)^T$ are

(a) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & -3 \\ -3 & 40 \end{bmatrix}$

(b) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & -6 \\ -6 & 20 \end{bmatrix}$

(c) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & 3 \\ 3 & 20 \end{bmatrix}$

(d) $\begin{pmatrix} 0 \\ 5 \end{pmatrix}, \begin{bmatrix} 5 & 6 \\ 6 & 40 \end{bmatrix}$

Qus. 52 The number of solutions of the equation $x^2 = 1$ in the ring $\mathbb{Z}/105\mathbb{Z}$ is

- (a) 0 (b) 2
(c) 4 (d) 8

Qus. 53 Let $f(z) = \exp\left(z + \frac{1}{z}\right)$, $z \in \mathbb{C} \setminus \{0\}$. The residue of f at $z = 0$ is

(a) $\sum_{l=0}^{\infty} \frac{1}{(l+1)!}$

(b) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)}$

(c) $\sum_{l=0}^{\infty} \frac{1}{l!(l+1)!}$

(d) $\sum_{l=0}^{\infty} \frac{1}{(l^2 + l)!}$

Qus. 54 Let p be a prime number. Let G be a group such that for each $g \in G$ there exists an $n \in \mathbb{N}$ such that $g^{p^n} = 1$. Which of the following statements is false?

- (a) If $|G| = p^6$, then G has a subgroup of index p^2
(b) If $|G| = p^6$, then G has at least five normal subgroups.
(c) Center of G can be infinite
(d) There exists G with $|G| = p^6$ such that G has exactly six normal subgroups

Qus. 55 Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be a locally Lipschitz function. Consider the initial value problem

$$\dot{x} = f(t, x), \quad x(t_0) = x_0 \quad \text{for } (t_0, x_0) \in \mathbb{R}^2.$$

Suppose that $J(t_0, x_0)$ represents the maximal interval of existence for the initial value problem. Which of the following statement is true?

- (a) $J(t_0, x_0) = \mathbb{R}$
(b) $J(t_0, x_0)$ is an open set
(c) $J(t_0, x_0)$ is a closed set
(d) $J(t_0, x_0)$ could be an empty set

Qus. 56 Let f be an entire function that satisfies

$$|f(z)| \leq e^y \quad \text{for all } z = x + iy, \text{ where } x, y \in \mathbb{R}.$$

Which of the following statements is true?

- (a) $f(z) = ce^{-iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(b) $f(z) = ce^{iz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(c) $f(z) = e^{-ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$
(d) $f(z) = e^{ciz}$ for some $c \in \mathbb{C}$ with $|c| \leq 1$

Qus. 57 If $f(x)$ is a probability density on the real line, then which of the following is NOT a valid probability density?

- (a) $f(x+1)$ (b) $f(2x)$
(c) $2f(2x-1)$ (d) $3x^2 f(x^3)$

Qus. 58 Consider \mathbb{R} with the usual topology. Which of the following assertions is correct?

- (a) A finite set containing 33 elements has at least 3 different Hausdorff topologies
(b) Let X be a non-empty finite set with a Hausdorff topology. Consider $X \times X$ with the product topology. Then every function $f: X \times X \rightarrow \mathbb{R}$ is continuous
(c) Let X be a discrete topological space having infinitely many elements. Let $f: \mathbb{R} \rightarrow X$ be a continuous function and $g: X \rightarrow \mathbb{R}$ be any non-constant function. Then the range of $g \circ f$ contains at least 2 elements.

- (d) If a non-empty metric space X has a finite dense subset, then there exists a discontinuous function $f: X \rightarrow \mathbb{R}$

Qus. 59 Let A be a 3×3 matrix with real entries.

Which of the following assertions is false?

- (a) A must have a real eigenvalue
(b) If the determinant of A is 0, then is an eigenvalue of A
(c) If the determinant of A is negative and 3 is an eigenvalue of A , then A must have three real eigenvalues.
(d) If the determinant of A is positive and 3 is an eigenvalue of A , then A must have three real eigenvalues

Qus. 60 Let X_1, \dots, X_7 and Y_1, \dots, Y_9 be two random sample drawn independently from two populations with continuous CDFs F and G respectively. Consider the wald- Wolfowitz run test in the context of the following two sample testing problems. $H_0: F(x) = G(x) \forall x$ vs

$H_1: F(x) \neq G(x)$ for some x . If the random variable R denotes the total number of runs in the combined ordered arrangement of the two given sample, then which of the following is true?

- (a) $P_{H_0}(R=6) = \frac{28}{286}, P_{H_0}(R=9) = \frac{28}{143}$
(b) $P_{H_0}(R=6) = \frac{21}{286}, P_{H_0}(R=9) = \frac{15}{28}$
(c) $P_{H_0}(R=6) = \frac{21}{286}, P_{H_0}(R=9) = \frac{28}{143}$
(d) $P_{H_0}(R=6) = \frac{21}{286}, P_{H_0}(R=9) = \frac{15}{286}$

PART - C:-

Qus. 61 Suppose that $f: \mathbb{R}^n \rightarrow \mathbb{R}$, $n \geq 2$ is a C^2 function satisfying

$$f(y) \geq f(x) + \nabla(f)(x)(y-x) \text{ for every } x, y$$

in \mathbb{R}^n . Here ∇ denotes gradient. Which of the following statements are true?

- (a) f is constant
(b) f is convex

- (c) f is convex and bounded
(d) f is constant if f is bounded

Qus. 62 A cumulative hazard function $H(t)$ of a non-negative continuous random variable satisfies which of the following conditions?

- (a) $\lim_{t \rightarrow \infty} H(t) = \infty$
(b) $H(0) = 0$
(c) $H(1) = 1$
(d) $H(t)$ is a non decreasing function of t

Qus. 63 Let $\{X_i: 1 \leq i \leq 2n\}$ be independently and identically distributed normal random variables with mean μ and variance 1, and independent of a standard Cauchy random variable W . Which of the following statistics are consistent for μ ?

- (a) $n^{-1} \sum_{i=1}^n X_i$
(b) $n^{-1} \sum_{i=1}^{2n} X_i$
(c) $n^{-1} \sum_{i=1}^n X_{2i-1}$
(d) $n^{-1} \left(\sum_{i=1}^n X_i + W \right)$

Qus. 64 Let $f: \{z: |z| < 1\} \rightarrow \{z: |z| \leq 1/2\}$ be a holomorphic function such that $f(0) = 0$. Which of the following statements are true?

- (a) $|f(z)| \leq |z|$ for all z in $\{z: |z| < 1\}$
(b) $|f(z)| \leq \left| \frac{z}{2} \right|$ for all z in $\{z: |z| < 1\}$
(c) $|f(z)| \leq 1/2$ for all z in $\{z: |z| < 1\}$
(d) It is possible that $f(1/2) = 1/2$

Qus. 65 Consider the linear model $E(Y_1) = 2\beta_1 - \beta_2 - \beta_3$, $E(Y_2) = \beta_2 - \beta_4$, $E(Y_3) = \beta_2 + \beta_3 - 2\beta_4$ with uncorrelated and homoscedastic random error. Which of the

following linear parametric functions are estimable?

- (a) $16\beta_1 - 9\beta_2 - 7\beta_3$
- (b) $\beta_3 - \beta_4$
- (c) $57\beta_1 - 18\beta_2 - 13\beta_3 - 26\beta_4$
- (d) $23\beta_1 - 9\beta_2 - 10\beta_3 + 4\beta_4$

Qus. 66 Consider the multiple linear regression model $Y = X\beta + \epsilon$; where Y is $n \times 1$ observed data vector with $n > 5$; X is $n \times 5$ matrix of known constants with rank $(X) = 5$, $\beta = (\beta_0, \beta_1, \beta_2, \beta_3, \beta_4)^T$ and $\epsilon = (\epsilon_1, \dots, \epsilon_n)^T$, where ϵ_i for $i = 1, \dots, n$ are independent and identically $N(0, 1)$ random variables. Consider testing of alternative $H_1: H_0$ is not true. Which of the following statements are true?

- (a) The sum of squares residuals under H_0 follows a central χ^2 distribution with $(n-5)$ degrees of freedom.
- (b) The sum of squares residuals under H_0 follows a central χ^2 distribution with $(n-1)$ degrees of freedom
- (c) The test statistic follows a central F distribution with $(5, n-1)$ degrees of freedom
- (d) The test statistic follows a central F distribution with $(4, n-5)$ degrees of freedom

Qus. 67 Let A, B be two events in a discrete probability space with $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$. Which of the following are necessarily true.

- (a) If $\mathbb{P}(A|B) = 0$ then $\mathbb{P}(B|A) = 0$
- (b) If $\mathbb{P}(A|B) = 1$ then $\mathbb{P}(B|A) = 1$
- (c) If $\mathbb{P}(A|B) > \mathbb{P}(A)$ then $\mathbb{P}(B|A) > \mathbb{P}(B)$
- (d) If $\mathbb{P}(A|B) > \mathbb{P}(B)$ then $\mathbb{P}(B|A) > \mathbb{P}(A)$

Qus. 68 Consider the following statements:

- (A) Let f be a continuous function on $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x) dx$ converges. Then $\sum_{n \geq 1} f(n)$ converges
- (B) Let f be a function on $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x) dx$ converges. Then $\lim_{x \rightarrow \infty} f(x) = 0$
- (C) Let f be a continuous, decreasing function $[1, \infty)$ taking non-negative values such that $\int_1^\infty f(x) dx$ does not converge. Then $\sum_{n \geq 1} f(n)$ does not converge

Which of the following options are true?

- (a) (A) (B) and (C) all are true
- (b) Both (A) and (B) are false
- (c) (C) is true
- (d) (B) is true

Qus. 69 Let $(X_1, Y_1), \dots, (X_4, Y_4)$ be a random sample from a continuous bivariate distribution function $F_{X,Y}$ with marginal distribution of X and Y being F_X and F_Y respectively. In order to test the null hypothesis $H_0: X$ and Y are independent against the alternative $H_1: X$ and Y are positively, consider the Kendall sample correlation statistic

$$K = \sum_{i=1}^3 \sum_{j=i+1}^4 \psi((X_i, Y_i), (X_j, Y_j)) \text{ where}$$

$$\psi((a, b), (c, d)) = \begin{cases} 1, & \text{if } (d-b)(c-a) > 0 \\ -1, & \text{if } (d-b)(c-a) < 0 \end{cases}$$

Assuming no ties which of the following are true?

- (a) The test that rejects H_0 for $K \geq 4$ has size $1/4$
- (b) The test that rejects H_0 for $K \geq 4$ has size $1/6$

(c) $P_{H_0}(K=4)=3/24$

(d) $P_{H_0}(K=6)=1/12$

Qus. 70 Suppose that cars arrive at a petrol pump following a Poisson distribution at the rate of 10 per hour. The time to perform the refilling is exponentially distribution and the single available staff takes an average of 4 minutes to refill each car. Further assume that the cars leave immediately after refilling. Let α denote the probability of finding 3 or more cars waiting to refill and let β denote the mean number of cars in the queue. Which of the following statements are correct?

(a) $\alpha = \frac{8}{27}$

(b) $\beta = 1$

(c) $\beta - \alpha = \frac{46}{27}$

(d) $\alpha\beta = 3$

Qus. 71 Let $X = \prod_{n=1}^{\infty} [0,1]$, that is, the space of sequences $\{x_n\}_{n \geq 1}$ with $x_n \in [0,1]$, $n \geq 1$. Define the metric $d: X \times X \rightarrow [0, \infty)$ by

$$d(\{x_n\}_{n \geq 1}, \{y_n\}_{n \geq 1}) = \sup_{n \geq 1} \frac{|x_n - y_n|}{2^n}.$$

Which of the following statements are true?

- (a) The metric topology on X is finer than the product topology on X
- (b) The metric topology on X is coarser than the product topology on X
- (c) The metric topology on X is same as the product topology on X
- (d) The metric on X and the product topology on X are not comparable.

Qus. 72 Which of the following are true?

- (a) For $n \geq 1$, the sequence of functions $f_n: (0,1) \rightarrow (0,1)$ defined by $f_n(x) = x^n$ is uniformly convergent
- (b) For $n \geq 1$, the sequence of functions $f_n: (0,1) \rightarrow (0,1)$ defined by

$$f_n(x) = \frac{x^n}{\log(n+1)}$$
 is uniformly convergent

- (c) For $n \geq 1$, the sequence of functions

$$f_n: (0,1) \rightarrow (0,1) \text{ defined by } f_n(x) = \frac{x^n}{1+x^n}$$
 is uniformly convergent

- (d) For $n \geq 1$, the sequence of functions

$$f_n: (0,1) \rightarrow (0,1) \text{ defined by } f_n(x) = \frac{x^n}{1+n x^n}$$
 is not uniformly convergent

Qus. 73 Suppose $y(x)$ is an extremal of the following functional

$$J(y(x)) = \int_0^1 (y(x)^2 - 4y(x)y'(x) + 4y'(x)^2) dx$$

subject to $y(0)=1$ and $y'(0)=1/2$

Which of the following statements are true?

- (a) y is a convex function
- (b) y is concave function
- (c) $y(x_1+x_2) = y(x_1)y(x_2)$ for all x_1, x_2 in $[0,1]$
- (d) $y(x_1x_2) = y(x_1)y(x_2)$ for all $x_1, x_2 \in [0,1]$

Qus. 74 Let $f(z)$ be an entire function on \mathbb{C} which of the following statements are true?

- (a) $f(\bar{z})$ is an entire function
- (b) $\overline{f(z)}$ is an entire function
- (c) $\overline{f(\bar{z})}$ is an entire function
- (d) $\overline{f(z)} + f(\bar{z})$ is an entire function

Qus. 75 Let $\lambda_1 < \lambda_2$ be two real characteristic numbers for the following homogeneous integral equation: $\varphi(x) = \lambda \int_0^{2\pi} \sin(x+t) \varphi(t) dt$; and let $\mu_1 < \mu_2$ be two real characteristic numbers for the following homogeneous integral equation:

$$\psi(x) = \mu \int_0^{\pi} \cos(x+t) \psi(t) dt$$

Which of the following statements are true?

- (a) $\mu_1 < \lambda_1 < \lambda_2 < \mu_2$
 (b) $\lambda_1 < \mu_1 < \mu_2 < \lambda_2$
 (c) $|\mu_1 - \lambda_1| = |\mu_2 - \lambda_2|$
 (d) $|\mu_1 - \lambda_1| = 2|\mu_2 - \lambda_2|$

Qus. 76 Under which of the following conditions is the sequence $\{x_n\}$ of real numbers convergent?

- (a) The subsequences $\{x_{(2n+1)}\}, \{x_{2n}\}$ and $\{x_{3n}\}$ are convergent and have the same limit
 (b) The subsequences $\{x_{(2n+1)}\}, \{x_{2n}\}$ and $\{x_{3n}\}$ are convergent
 (c) The subsequences $\{x_{kn}\}_n$ are convergent for every $k \geq 2$
 (d) $\lim_n |x_{(n+1)} - x_n| = 0$

Qus. 77 Suppose under the null hypothesis $H, X \sim p$, where $p(x) = P(X=x) = 1/20$, $x \in \{1, 2, \dots, 20\}$ and under the alternative hypothesis $K, X \sim q$ where

$$q(x) = p(X=x) = \frac{x}{210}, x \in \{1, 2, \dots, 20\}.$$

Define two test functions ϕ and ψ for testing H against K such that

$$\phi(x) = \begin{cases} 1 & \text{if } x \leq 2 \\ 0 & \text{otherwise} \end{cases} \text{ and}$$

$$\psi(x) = \begin{cases} 1 & \text{if } x \geq 19 \\ 0 & \text{otherwise} \end{cases}$$

Which of the following statements are true?

- (a) Size of the test ϕ is 0.1
 (b) Size of the test ψ is 0.05
 (c) $\{\text{Power of the test } \psi\} \geq 0.05$
 (d) $\{\text{Power of the test } \psi\} > \{\text{Power of the test } \phi\}$

Qus. 78 Let $y(x)$ and $z(x)$ be the stationary functions [extremals] of the variational problem

$$J(y(x), z(x)) = \int_0^1 [(y')^2 + (z')^2 + y'z'] dx$$

subject to

$$y(0) = 1, y(1) = 0, z(0) = -1, z(1) = 2$$

Which of the following statements are correct?

- (a) $z(x) + 3y(x) = 2$ for $x \in [0, 1]$
 (b) $3z(x) + y(x) = 2$ for $x \in [0, 1]$
 (c) $y(x) + z(x) = 2x$ for $x \in [0, 1]$
 (d) $y(x) + z(x) = x$ for $x \in [0, 1]$

Qus. 79 Which of the following statements are true?

- (a) Maximum likelihood estimator may not be unique
 (b) A complete statistic is always sufficient
 (c) A sufficient statistic may not be complete
 (d) Any function of a sufficient statistic is always sufficient

Qus. 80 Let G be a group of order 2023. Which of the following statements are true?

- (a) G is an Abelian group
 (b) G is a cyclic group
 (c) G is a simple group
 (d) G is not a simple group

Qus. 81 A point particle having unit mass is moving in x, y plane having the Lagrangian as

$$\text{follows } L = \dot{x} \dot{y} - 2x^2 - 2y^2$$

What are the possible values of pr (conjugate momentum to radial coordinate in plane polar coordinate)?

- (a) \dot{r}
 (b) $\dot{r} \sin 2\theta + r \dot{\theta} \cos 2\theta$
 (c) $\dot{r} \sin \theta + r \dot{\theta} \cos \theta$
 (d) $2\dot{r} \sin \theta + r \dot{\theta} \cos \theta$

Qus. 82 Let $f \in C^1(\mathbb{R})$ be bounded. Let us consider the initial-value problem

$$(P) \begin{cases} x'(t) = f(x(t)), t > 0 \\ x(0) = 0 \end{cases}$$

Which of the following statements are true?

- (a) (P) has solutions (s) defined for all $t > 0$
- (b) (P) has a unique solution
- (c) (P) has infinitely many solutions
- (d) The solution (s) of (P) is/are Lipschitz

Qus. 83 Let X and Y be independent Poisson random variables with parameters 2 and 3 respectively. Which of the following statements are correct?

- (a) $Var(X | X + Y = 2) = \frac{12}{25}$
- (b) $E\left(\frac{2}{1+x} | X + Y = 2\right) = \frac{98}{3}$
- (c) $P(X^2 = 0 | X + Y = 2) = e^{-2} + \frac{9}{25}(1 - e^{-2})$
- (d) $X | Y = 3 \sim \text{Binomial}(3, 2)$

Qus. 84 Let G_1 and G_2 be two groups and $\varphi: G_1 \rightarrow G_2$ be a surjective group homomorphism. Which of the following statements are true?

- (a) If G_1 is cyclic then G_2 is cyclic
- (b) If G_1 is Abelian then G_2 is Abelian
- (c) If H is a subgroup of G_1 then $\varphi(H)$ is a subgroup of G_2
- (d) If N is a normal subgroup of G_1 then $\varphi(N)$ is a normal subgroup of G_2

Qus. 85 Let B be a 3×5 matrix with entries from

\mathbb{Q} . Assume that $\{v \in \mathbb{R}^5 | Bv = 0\}$ is a three dimensional real vector space. Which of the following statements are true?

- (a) $\{v \in \mathbb{Q}^5 | Bv = 0\}$ is a three-dimensional vector space over \mathbb{Q}
- (b) The linear transformation $T: \mathbb{Q}^3 \rightarrow \mathbb{Q}^5$ given by $T(v) = B^t v$ is injective
- (c) The column span of B is two-dimensional
- (d) The linear transformation $T: \mathbb{Q}^3 \rightarrow \mathbb{Q}^3$ given

by $T(v) = B B^t v$ is injective

Qus. 86 Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation satisfying $T^3 - 3T^2 = -2I$, where

$I: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is the identity transformation. Which of the following statements are true?

- (a) \mathbb{R}^3 must admit a basis B_1 such that the matrix of T with respect to B_1 is symmetric
- (b) \mathbb{R}^3 must admit a basis B_2 such that the matrix of T with respect to B_2 is upper triangular
- (c) \mathbb{R}^3 must contain a non-zero vector v such that $Tv = v$
- (d) \mathbb{R}^3 must contain two linearly independent vectors v_1, v_2 such that $Tv_1 = v_1$ and $Tv_2 = v_2$

Qus. 87 Define a function $f: \mathbb{R} \rightarrow \mathbb{R}$ by

$$f(x) = \begin{cases} \sin(\pi/x) & \text{when } x \neq 0 \\ 0 & \text{when } x = 0 \end{cases}$$

On which of the following subsets of \mathbb{R} , the restriction of f is continuous function?

- (a) $[-1, 1]$
- (b) $(0, 1)$
- (c) $\{0\} \cup \{(1/n): n \in \mathbb{N}\}$
- (d) $\{1/2^n: n \in \mathbb{N}\}$

Qus. 88 Which of the following statements are correct?

- (a) The set of open right half planes is a basis for the usual (Euclidean) topology on \mathbb{R}^2
- (b) The set of lines parallel to Y-axis is a basis for the dictionary order topology on \mathbb{R}^2
- (c) The set of open rectangles is a basis for the usual (Euclidean) topology on \mathbb{R}^2
- (d) The set of line segments (without end points) parallel to Y-axis is a basis for the dictionary order topology on \mathbb{R}^2

- Qus. 89** Let μ denote the Lebesgue measure on \mathbb{R} and μ^* be the associated Lebesgue outer measure. Let A be a non-empty subset of $[0, 1]$. Which of the following statements are true?
- If both interior and closure of A have the same outer measure, then A is Lebesgue measurable.
 - If A is open, then A is Lebesgue measurable and $\mu(A) > 0$
 - If A is not Lebesgue measurable, then the set of irrationals in A must have positive outer measure.
 - If $\mu^*(A) = 0$ then A has empty interior

Qus. 90 Let $u = u(x, y)$ be the solution to the following Cauchy problem $u_x + u_y = e^u$ for $(x, y) \in \mathbb{R} \times \left(0, \frac{1}{e}\right)$ and $u(x, 0) = 1$ for $x \in \mathbb{R}$. Which of the following statements are true?

- $u\left(\frac{1}{2e}, \frac{1}{2e}\right) = 1$
- $u_x\left(\frac{1}{2e}, \frac{1}{2e}\right) = 0$
- $u_y\left(\frac{1}{4e}, \frac{1}{4e}\right) = \log 4$
- $u_y\left(0, \frac{1}{4e}\right) = \frac{4e}{3}$

- Qus. 91** Let V be the vector space of all polynomials in one variable of degree at most 10 with real coefficients. Let W_1 be the subspace of V consisting of polynomials of degree at most 5 and let W_2 be the subspace of V consisting of polynomials such that the sum of their coefficients is 0. Let W be the smallest subspace of V containing both W_1 and W_2 . Which of the following statements are true?
- The dimension of W is at most 10
 - $W = V$
 - $W_1 \subset W_2$
 - The dimension of $W_1 \cap W_2$ is at most 5

Qus. 92 Let V be the real vector space of real polynomials in one variable with degree less than or equal to 10 (including the zero polynomial). Let $T: V \rightarrow V$ be the linear map defined by $T(p) = p'$, where p' denotes the derivative of p . Which of the following statements are correct?

- $\text{rank}(T) = 10$
- $\text{determinant}(T) = 0$
- $\text{trace}(T) = 0$
- All the eigenvalues of T are equal to 0

Qus. 93 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as

$$f(x) = \frac{1}{4} + x - x^2, \text{ Given } a \in \mathbb{R}, \text{ let us define}$$

the sequence $\{x_n\}$ by $x_0 = a$ and $x_n = f(x_{n-1})$ for $n \geq 1$.

Which of the following statements are true?

- If $a = 0$, then the sequence $\{x_n\}$ converges to $\frac{1}{2}$
- If $a = 0$, then the sequence $\{x_n\}$ converges to $-\frac{1}{2}$
- The sequence $\{x_n\}$ converges for every $a \in \left(-\frac{1}{2}, \frac{3}{2}\right)$, and it converges to $\frac{1}{2}$
- If $a = 0$, then the sequence $\{x_n\}$ does not converge

Qus. 94 Let $Y_i = a + \beta x_i + \varepsilon_i, i = 1, 2, 3$, where x_i 's are fixed covariates, α and β are unknown parameters and ε_i 's are independently and identically distributed normal random variables with mean 0 and variance $\sigma^2 > 0$. Given the following observations.

x_i	1	2	3
y_i	2.1	3.9	6

Which of the following statements are true?

- (a) Maximum likelihood estimate of α is 0.1
- (b) Least square estimate of α is 0.1
- (c) Best linear unbiased estimate of α is 0.1
- (d) Maximum likelihood estimate of $\frac{\beta}{\alpha}$ is 19.5

Qus. 95 Consider the following initial value problem (IVP) $\frac{du}{dt} = t^2 u^{\frac{1}{5}}, u(0) = 0$

Which of the following statements are correct?

- (a) The function $g(t, u) = t^2 u^{\frac{1}{5}}$ does not satisfy the Lipschitz's condition with respect to u in any neighbourhood of $u = 0$
- (b) There is no solution for the IVP
- (c) There exist more than one solution for the IVP
- (d) The function $g(t, u) = t^2 u^{\frac{1}{5}}$ satisfies the Lipschitz's condition with respect to u in some neighbourhood of $u = 0$ and hence there exists a unique solution for the IVP

Qus. 96 Let $n \geq 1$ be a positive integer and S_n the symmetric group on n symbols. Let $\Delta = \{(g, g) : g \in S_n\}$. Which of the following statements are necessarily true?

- (a) The map $f : S_n \times S_n \rightarrow S_n$ given by $f(a, b) = ab$ is a group homomorphism
- (b) Δ is a subgroup of $S_n \times S_n$
- (c) Δ is a normal subgroup of $S_n \times S_n$
- (d) Δ is a normal subgroup of $S_n \times S_n$, If n is a prime number

Qus. 97 Suppose X_1, X_2, \dots independent and identically distributed $N(0, 1)$ random variable and $Y_n = X_1^4 + X_2^4 + \dots + X_n^4$. Which of the following probabilities converge to $\frac{1}{2}$ as $n \rightarrow \infty$?

- (a) $\mathbb{P}\{Y_n \in [0, 2n]\}$
- (b) $\mathbb{P}\{Y_n \in [n, 3n]\}$
- (c) $\mathbb{P}\{Y_n \in [2n, 4n]\}$
- (d) $\mathbb{P}\{Y_n \in [3n, 5n]\}$

Qus. 98 Let E be a finite algebraic Galois extension of F with Galois group G . Which of the following statements are true?

- (a) There is an intermediate field K with $K \neq F$ and $K \neq E$ such that K is a Galois extension of F
- (b) If every proper intermediate field K is a Galois extension of F then G is abelian
- (c) If E has exactly three intermediate fields including F and E then G is abelian
- (d) If $[E : F] = 99$ then every intermediate field is a Galois extension of F

Qus. 99 Let $K \in C([0, 1] \times [0, 1])$ satisfy

$|K(x, y)| < 1$ for all $x, y \in [0, 1]$. For every $g \in C[0, 1]$, let us consider the integral equation

$$(P_g)f(x) + \int_0^1 K(x, y)f(y)dy = g(x)$$

for all $x \in [0, 1]$.

Which of the following statements are true?

- (a) There exists a $g \in C[0, 1]$ for which (P_g) has no solution in $C[0, 1]$
- (b) (P_g) has a solution in $C[0, 1]$ for infinitely many $g \in C[0, 1]$
- (c) The solution of (P_g) in $C[0, 1]$ is unique if $g \in C^1[0, 1]$
- (d) There exists a $g \in C[0, 1]$ for which (P_g) has infinitely many solutions in $C[0, 1]$

Qus. 100 Define $f : \mathbb{R}^4 \rightarrow \mathbb{R}$ by

$$f(x, y, z, w) = xw - yz.$$

Which of the following statements are true?

- (a) f is continuous
- (b) If $U = \{(x, y, z, w) \in \mathbb{R}^4 : xy + zw = 0, x^2 + z^2 = 1, y^2 + w^2 = 1\}$, then f is uniformly continuous on U
- (c) If $V = \{(x, y, z, w) \in \mathbb{R}^4 : x = y, z = w\}$, then f is uniformly continuous on V
- (d) If $W = \{(x, y, z, w) \in \mathbb{R}^4 : 0 \leq x + y + z + w \leq 1\}$ then f is unbounded on W

Qus. 101 Let $n \geq 2$ be a positive integer. Consider a Markov Chain on the state space $\{1, 2, \dots, n\}$ with a given transition probability matrix P . Let I_n denote the identity matrix of order n . Which of the following statements are necessarily true?

- (a) At least one state is recurrent
- (b) At least one state is transient
- (c) $-\frac{1}{3}I_n + \frac{4}{3}P$ is also a transition probability matrix of some Markov chain
- (d) 5 is an eigenvalue of $I_n + 3P + P^2$

Qus. 102 Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = x^2 - y^3$.

Which of the following statements are true?

- (a) There is no continuous real-valued function g defined on any interval of \mathbb{R} containing 0 such that $f(x, g(x)) = 0$
- (b) There is exactly one continuous real-valued function g defined on an interval of \mathbb{R} containing 0 such that $f(x, g(x)) = 0$
- (c) There is exactly one differentiable real-valued function g defined on an interval of \mathbb{R} containing 0 such that $f(x, g(x)) = 0$
- (d) There are two distinct differential real-valued functions g on an interval of \mathbb{R} containing 0 such that $f(x, g(x)) = 0$.

Qus. 103 Let a continuous random variable X follow uniform $(-1, 1)$. Define $Y = X^2$. Which

of the following are not true for X and Y ?

- (a) They are independent and uncorrelated
- (b) They are independent but correlated
- (c) They are not independent but correlated
- (d) They are neither independent nor correlated

Qus. 104 Which of the following statements are true for an arbitrary normed linear space U ?

- (a) Every bounded linear functional from U to \mathbb{R} is continuous
- (b) U is isomorphic to its double-dual U^{**}
- (c) For every $x \in U$, we have $\|x\| = \sup_{\|f\| \leq 1} |f(x)|$, where f denotes elements of U^*
- (d) The closed unit ball in U is compact

Qus. 105 Let X_i for $i = 1, 2, \dots, 2n$, $n \geq 1$, be independent random variables each distributed as $N(0, 1)$. Which of the following statements are correct?

- (a) $(X_1 + \dots + X_n - X_{n+1} - \dots - X_{2n}) / 2n \sim N(0, 2)$
- (b) $(X_1 - X_2)^2 + (X_3 - X_4)^2 + \dots + (X_{2n-1} - X_{2n})^2 \sim 2 \chi_n^2$
- (c) $E\left[\max(|X_1|, |X_{n+1}|)\right] = \frac{2}{\sqrt{\pi}}$
- (d) $E\left[\max(|X_1|, |X_{n+1}|)\right] = \frac{4}{\sqrt{\pi}}$

Qus. 106 Consider the following two sequences $\{a_n\}$ and $\{b_n\}$ given by

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n},$$

$$b_n = \frac{1}{n}$$

Which of the following statements are true?

- (a) $\{a_n\}$ converges to $\log 2$, and has the same convergence rate as the sequence $\{b_n\}$

- (b) $\{a_n\}$ converges $\log 4$, and has the same convergence rate as the sequence $\{b_n\}$
- (c) $\{a_n\}$ converges to $\log 2$, but does not have the same convergence rate as the sequence $\{b_n\}$
- (d) $\{a_n\}$ does not converge

Qus. 107 Let $\{x_n\}$ be a sequence of positive real numbers. If $\sigma_n = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$ then

which of the following are true? (Here \limsup denotes the limit supremum of a sequence)

- (a) If $\limsup\{x_n\} = \ell$ and $\{x_n\}$ is decreasing, then $\limsup\{\sigma_n\} = \ell$
- (b) $\limsup\{x_n\} = \ell$ if and only if $\limsup\{\sigma_n\} = \ell$
- (c) If $\limsup\left\{n\left(\frac{x_n}{x_{(n+1)}}\right)\right\} < 1$, then $\sum_n x_n$ is convergent
- (d) If $\limsup\left\{n\left(\frac{x_n}{x_{(n+1)}} - 1\right)\right\} < 1$, then $\sum_n x_n$ is divergent

Qus. 108. Let $u: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x u + 2\partial_y u = 0 & \text{for } (x, y) \in \mathbb{R}^2 \\ u(x, y) = \sin(x) & \text{for } y = 3x + 1, x \in \mathbb{R} \end{cases}$$

Let $v: \mathbb{R}^2 \rightarrow \mathbb{R}$ be the solution to the Cauchy problem:

$$\begin{cases} \partial_x v + 2\partial_y u = 0 & \text{for } (x, y) \in \mathbb{R}^2 \\ v(x, 0) = \sin(x) & \text{for } x \in \mathbb{R} \end{cases}$$

Let $\delta = [0, 1] \times [0, 1]$

Which of the following statements are true?

- (a) u changes sign in the interior of δ
- (b) $u(x, y) = v(x, y)$ along a line in δ

- (c) v changes sign in the interior of δ
- (d) v vanishes along a line in δ

Qus. 109 Which of the following are maximal ideal of $\mathbb{Z}[x]$?

- (a) Ideal generated by 2 and $(1 + X^2)$
- (b) Ideal generated by 2 and $(1 + X + X^2)$
- (c) Ideal generated by 3 and $(1 + X^2)$
- (d) Ideal generated by 3 and $(1 + X + X^2)$

Qus. 110 Let V be a finite dimensional real vector space and T_1, T_2 be two nilpotent operators on V . Let $W_1 = \{v \in V : T_1(v) = 0\}$ and $W_2 = \{v \in V : T_2(v) = 0\}$. Which of the following statements are false?

- (a) If T_1 and T_2 are similar, then W_1 and W_2 are isomorphic vector spaces.
- (b) If W_1 and W_2 are isomorphic vector spaces, then T_1 and T_2 have the same minimal polynomial
- (c) If $W_1 = W_2 = V$, then T_1 and T_2 are similar
- (d) If W_1 and W_2 are isomorphic, then T_1 and T_2 have the same characteristic polynomial.

Qus. 111 Which of the following statements are correct?

- (a) If G is a group order 244, then G contains a unique subgroup of order 27
- (b) If G is a group of order 1694, then G contains a unique subgroup of order 121
- (c) There exists a group of order 154 which contains a unique subgroup of order 7
- (d) There exists a group of order 121 which contains two subgroups of order 11

Qus. 112 Suppose A is a 5×5 block diagonal real

matrix with diagonal blocks $\begin{pmatrix} e & 1 \\ 0 & e \end{pmatrix}$,

$$\begin{pmatrix} e & 1 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{pmatrix}$$

Which of the following statements are true?

- (a) The algebraic multiplicity of e in A is 5
- (b) A is not diagonalisable
- (c) The geometric multiplicity of e in A is 3
- (d) The geometric multiplicity of e in A is 2

Qus. 113 Suppose X_1, X_2, \dots, X_n are independently and identically distributed $N(0, \tau^{-2})$ random variables, for $\tau^{-2} > 0$. Let the prior distribution on τ^2 have density $\pi(\tau^{-2}) \propto (1/\tau^2)^\alpha$ for some $\alpha > 0$. Which of the following are true?

- (a) The prior distribution is an improper distribution for $\alpha > 0$
- (b) The posterior distribution is a proper distribution for all $\alpha > 0$
- (c) Under a squared error loss, the generalized

Bayes estimator of τ^2 is $\frac{n/2 - \alpha}{\sum_{i=1}^n x_i^2 / 2}$

- (d) The posterior distribution is proper for $\alpha = 1$

Qus. 114 Consider the following quadratic forms over \mathbb{R}

- (A) $6X^2 - 13XY + 6Y^2$
- (B) $X^2 - XY + 2Y^2$
- (C) $X^2 - XY - 2Y^2$

Which of the following statements are true?

- (a) Quadratic forms (A) and (B) are equivalent
- (b) Quadratic forms (A) and (C) are equivalent
- (c) Quadratic forms (B) is positive definite
- (d) Quadratic forms (C) is positive definite

Qus. 115 Suppose that X_1, \dots, X_n, X_{n+1} is a random sample of size $n+1$, where $p > 2$ and $n > p+3$, from a multivariate normal population, $N_p(\mu, \Sigma)$; $\mu \in \mathbb{R}^p$ and $\Sigma > 0$. Let

$$\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i \text{ and}$$

$$(n-1)S = \sum_{i=1}^n (X_i - \bar{X}_n)(X_i - \bar{X}_n)^T.$$

Which of the following are correct?

- (a) $(\bar{X}_n - X_{n+1})^T S^{-1} (\bar{X}_n - X_{n+1}) \sim \frac{p(n^2-1)}{n(n-p)} F_{p, n-p}$

$$(b) \quad E(\bar{X}_n^T S \bar{X}_n) = \text{trace}\left(\frac{\Sigma^2}{n}\right) + \mu^T \Sigma \mu$$

$$(c) \quad E(S^{-1}) = \frac{n-1}{n-p-2} \Sigma^{-1}$$

$$(d) \quad (\bar{X}_n - X_{n+1})^T \Sigma^{-1} (\bar{X}_n - X_{n+1}) \sim \frac{n+1}{n} \chi_p^2$$

Qus. 116 Let X_1 and X_2 be two independent random variables such that X_1 follows a gamma distribution with mean 10 and variance 10, and $X_2 \sim N(3, 4)$. Let f_1 and f_2 denote the density functions of X_1 and X_2 , respectively.

Define a new random variable Y so that for $y \in \mathbb{R}$, it has density function

$$f(y) = 0.4 f_1(y) + q f_2(y)$$

Which of the following are true?

$$(a) \quad q = 0.6$$

$$(b) \quad E[Y] = 5.8$$

$$(c) \quad \text{Var}(Y) = 3.04$$

$$(d) \quad Y = 0.4X_1 + qX_2$$

Qus. 117 Let us consider the following two initial

$$\text{value problem } (P) \begin{cases} x'(t) = \sqrt{x(t)}, t > 0 \\ x(0) = 0 \end{cases}$$

$$\text{and } (Q) \begin{cases} y'(t) = -\sqrt{y(t)}, t > 0 \\ y(0) = 0 \end{cases}$$

Which of the following statements are true?

- (a) (P) has a unique solution in $[0, \infty)$
- (b) (Q) has a unique solution in $[0, \infty)$
- (c) (P) has infinitely many solutions in $[0, \infty)$
- (d) (Q) has infinitely many solutions in $[0, \infty)$

Qus. 118 Let $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ be the open unit disc and C the positively oriented boundary $\{|z| = 1\}$. Fix a finite set $\{z_1, z_2, \dots, z_n\} \subset \mathbb{D}$ of distinct points and consider the polynomial $g(z) = (z - z_1)(z - z_2) \dots (z - z_n)$ of degree n . Let f be a holomorphic function in an open neighbourhood of \mathbb{D} and define

$$P(z) = \frac{1}{2\pi i} \int_C f(\zeta) \frac{g(\zeta) - g(z)}{(\zeta - z)g(\zeta)} d\zeta$$

Which of the following statements are true?

- (a) P is a polynomial of degree n
- (b) P is a polynomial of degree $n-1$
- (c) P is a rational function on \mathbb{C} with poles at z_1, z_2, \dots, z_n
- (d) $P(z_j) = f(z_j)$ for $j = 1, 2, \dots, n$

Qus. 119 Let V be an inner product space and let $v_1, v_2, v_3 \in V$ be an orthogonal set of vectors. Which of the following statements are true?

- (a) The vectors $v_1 + v_2 + 2v_3, v_2 + v_3 + 3v_3$ can be extended to a basis of V
- (b) The vectors $v_1 + v_2 + 2v_3, v_2 + v_3, v_2 + 3v_3$ can be extended to an orthogonal basis of V
- (c) The vectors $v_1 + v_2 + 2v_3, v_2 + v_3, 2v_1 + v_2 + 3v_3$ can be extended to a basis of V
- (d) The vectors $v_1 + v_2 + 2v_3, 2v_1 + v_2 + v_3, 2v_1 + v_2 + 3v_3$ can be extended to a basis of V

Qus. 120 Let $D = \{z \in \mathbb{C} : |z| < 1\}$. Consider the following statements.

- (A) $f: D \rightarrow D$ be a holomorphic function. Suppose α, β are distinct complex numbers in D such that $f(\alpha) = \alpha$ and $f(\beta) = \beta$. Then $f(z) = z$ for all $z \in D$
- (B) There does not exist a bijective holomorphic function from D to the set of all complex numbers whose imaginary part is positive.
- (C) $f: D \rightarrow D$ be a holomorphic function. Suppose $\alpha \in D$ be such that $f(\alpha) = \alpha$ and $f'(\alpha) = 1$. Then $f(z) = z$ for all $z \in D$.

Which of the following options are true?

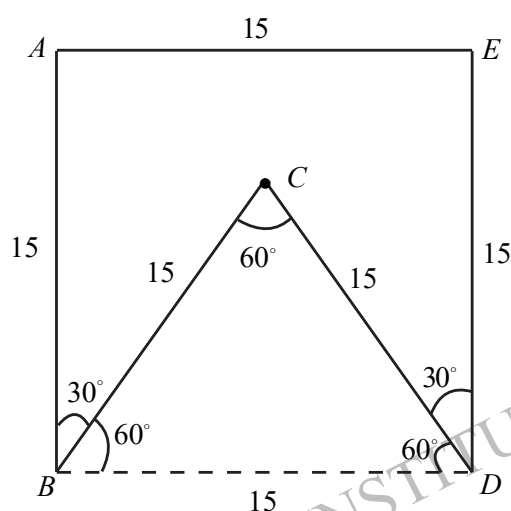
- (a) (A), (B) and (C) are all true
- (b) (A) is true
- (c) Both (A) and (B) are false
- (d) Both (A) and (C) are true.

CSIR NET SOLUTION

07-June-2023

PART "A"

Q.1 Ans (2)



In the given figure BCD is equilateral triangle
so $BC = CD = DB = 15$ &

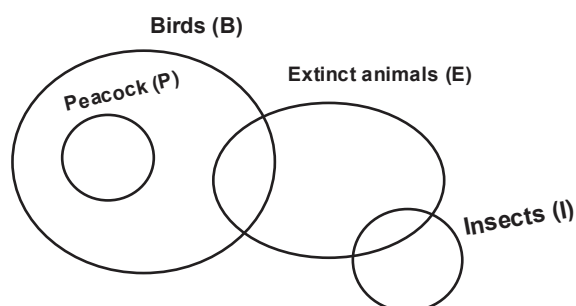
$$AB = DE = AE = 15$$

So perimeter of the field is

$$AE + ED + DC + CB + BA =$$

$$15 + 15 + 15 + 15 + 15 = 75$$

Q.2 Ans (3)



Venn diagram of Insects, Birds, Extinct animals and Peacocks.

Q.3 Ans (1)

Per capita GDP from 2010-2022 has increment followings

$$(i) \text{ For A : } \frac{115}{150} \text{ to } \frac{324}{160}$$

$$(ii) \text{ For B : } \frac{1675}{1200} \text{ to } \frac{2622}{1400}$$

$$(iii) \text{ For C : } \frac{177}{180} \text{ to } \frac{264}{220}$$

Ranking from high to low is
A, B, C

Q.4 Ans (3)

On first day batteries B_1, B_2, B_3 & B_4 will be,
on second day batteries B_5, B_6, B_1 & B_2 will
be used and on third day batteries
 B_5, B_6, B_3 & B_4 will be used.
So, no. of days = 3

Q.5 Ans (4)

Interest paid for 3 years with compound

interest rate of 10% will be $P \left(1 + \frac{r}{100} \right)^n - P =$

$$P \left[\left(1 + \frac{r}{100} \right)^n - 1 \right] =$$

$$150000 \left[\left(1 + \frac{10}{100} \right)^3 - 1 \right] =$$

$$150000 \left[\left(\frac{11}{10} \right)^3 - 1 \right] = \frac{150000 \times 331}{1000}$$

$$= 331 \times 150 = 49650 \text{ Rs.}$$

Q. 6 Ans (2)

Let the seven people be A,B,C,D,E,F & G then
A & B are siblings C & D are siblings and E,F
& G are siblings.

$$\boxed{AB}, \boxed{CD} \& \boxed{EFG}$$

Now number of ways to choose 2 persons from
these 7 persons will be 7C_2 and in favourable

Case for A there is 5 options

for B there is 5 options

for C there is 3 options

& for D there is 3 options

So required probability =

$$\frac{5+5+3+3}{{}^7C_2} = \frac{16}{21}$$

Q. 7 Ans (4)

Before blank number of E is equal to 16, so if
blank is filled by NINETEEN then number of
E in whole paragraph will be 19.

Q. 8 Ans (4)

If $a > b$ then

$$(ab)^2 - (ba)^2 = (10a+b)^2 - (10b+a)^2 =$$

$99a^2 - 99b^2 = 99(a^2 - b^2)$ which is divisible
by 11.

Q. 9 Ans (4)

If number of datas in A & B be n_1 & n_2 re-
spectively and both have mean u then sum
of all elements of A & B combined will be
 $n_1u + n_2u$ so mean of combined data will be

$$\frac{n_1u + n_2u}{(n_1 + n_2)} = u \text{ which is same as that of mean}$$

of data set A.

Q. 10 Ans (2)

Let A have coins of 1,2,5 & 10 equal to
 $3t, 2t, 2t$ & t respectively and B have coins of
1,2,5 & 10 equal to $4k, 3k, 2k$ & k respectively
then.

For A total amount is

$$270 = 3t \times 1 + 2t \times 2 + 2t \times 5 + t \times 10$$

$$\Rightarrow 270 = 27t \Rightarrow t = 10$$

Also, number of coins are equal so

$$3t + 2t + 2t + t = 4k + 3k + 2k + k$$

$$\Rightarrow 8k = 10k \& t = 10$$

$$8 \times 10 = 10k \Rightarrow k = 8$$

So, amount with B is

$$4k \times 1 = 3k \times 2 + 2k \times 5 + k \times 10$$

$$= (30)k = 30 \times 8 = 240$$

Q. 11 Ans (2)

If distance covered is d , actual speed is
 V km/hr. then

$$\frac{d}{V\left(1 + \frac{20}{100}\right)} = \frac{d}{V} - 2 \quad (1)$$

$$\& \frac{d}{V\left(1 - \frac{20}{100}\right)} = \frac{d}{V} + 3 \quad (2)$$

Let $\frac{d}{V} = t$ (normal travel time)

$$\frac{5}{6}t = t - 2 \Rightarrow \frac{1}{6}t = 2 \Rightarrow t = 12$$

From (1)

Q. 12 Ans (4)

If x answers are correctly answered, y an-
swers are incorrectly answered then

$10 - (x + y)$ are not attempted.

Hence marks will be $2x - y$ & $x + y \leq 10$

which cannot give $2x - y = 19$ as $2x - y = 19$

$\Rightarrow x = 10$ & $y = 1$ & thus $x + y = 11$ which is
impossible.

Q. 13 Ans (3)

Minimum probability that A & B will contra-
dict each other is $P(AB^C) + P(A^CB) =$

$$P(A)P(B^C) + P(A^C)P(B) =$$

$$\left(\frac{30}{100}\right)\left(1 - \frac{40}{100}\right) + \left(1 - \frac{30}{400}\right)\left(\frac{40}{100}\right) =$$

$$(0.3)(0.6) + (0.7)(0.4) =$$

$$0.18 + 0.28 = 0.46$$

Q. 14 Ans (1)

Let x be the present age of daughter then
 $14x$ will be present age of mother.
 After 4 years ratio of age of mother and daughter will be 16: 3

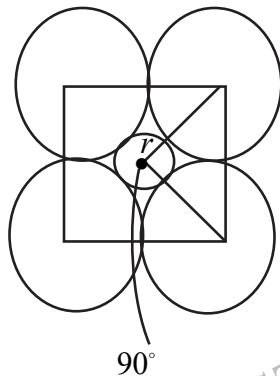
$$\text{so } \frac{16}{3} = \frac{14x+4}{x+4} \Rightarrow 16(x+4) = (14x+4) \times 3$$

$$\Rightarrow 16x + 64 = 42x + 12 \Rightarrow 26x = 52$$

$$\Rightarrow x = 2$$

So, present age of daughter is 2 & present age of mother is $14 \times 2 = 28$ so at time of birth of daughter age of mother was $28 - 2 = 26$ years

Q. 15 Ans (1)



$$(1+r)^2 + (1+r^2) = (1+1)^2$$

$$\Rightarrow 2(1+r)^2 = 4$$

$$\Rightarrow (1+r)^2 = 2$$

$$\Rightarrow 1+r = \sqrt{2}$$

$$\Rightarrow r = \sqrt{2} - 1$$

Now when spheres are stacked in pyramidal form then height of pyramid is $2 + 1 + (\sqrt{2} - 1) = 2 + \sqrt{2}$ here, 2 is diameter of upper sphere ; 1 = radius of lower sphere & $\sqrt{2} - 1$ = radius of cavity between upper sphere & radius of lower sphere.

Q. 16 Ans (4)

Let k be the constant of probability then
 $2k + 3k + 4k + 5k = 28$

$$\Rightarrow 14k = 28 \Rightarrow k = 2$$

So, number of windows of size 4ft. is 8 and number of windows of size 5ft. is 10.
 To escape window should be of size at least 4ft.
 So, number of windows for escaping is $8 + 10 = 18$.

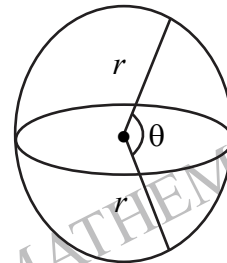
Q. 17 Ans (1)

$$\therefore \text{Var}(aX+b) = a^2 \text{Var}(X)$$

$$\therefore \text{Var}(3X+2) = 3^2 \text{Var}(X) = 3^2 \sigma^2 = (3\sigma)^2$$

$$\therefore \text{S.D.}(3X+2) = \sqrt{(3\sigma)^2} = 3\sigma$$

Q. 18 Ans ()



Let θ be the angle between A & B then distance between A & B is $r\theta$.

$$\text{At } r = 10 ; r\theta = 25 \Rightarrow \theta = \frac{5}{2}$$

$$\text{So, at } r = 50 ; r\theta = 50 \times \frac{5}{2} = 125$$

Q. 19 Ans (3)

Number of handshakes in the party

$$= 45C_2 - 40C_2 = \frac{45 \times 44}{2} - \frac{40 \times 39}{2} =$$

$$990 - 780 = 210$$

Q. 20 Ans (3)

Number of assembly seats

$$= 30 + 25 + 20 + 10 + 4 + 9 = 98$$

For majority number of seats should be more

$$\text{then } 50\% \text{ i.e. more than } 98 \times \frac{50}{100} = 49 \text{ i.e.}$$

at least 50 seats.

So, 1) No party has majority

2) $A + C = 30 + 20 > 49$ so $A + C$ can form government.

4) MLA from any party can become chief minister.

1, 2 & 4 are correct.

3) $A + D$ independent

$= 30 + 10 + 9 = 49 < 50$ so they cannot form government.

So, (3) is incorrect.

PART "B"

Q. 21 Ans (3)

Let $x = (x_1, x_2, \dots, x_n)$, $y = (y_1, y_2, \dots, y_n)$ &

$z = (z_1, z_2, \dots, z_n)$ from R^n &

$$\langle x, y \rangle = \sum_{j=1}^n j^3 x_j y_j$$

$$\Rightarrow \langle \overline{y}, x \rangle = \sum_{j=1}^n j^3 y_j x_j = \langle x, y \rangle$$

(Conjugate symmetry property)

$$\langle ax + by, z \rangle = \sum_{j=1}^n j^3 (ax_j + by_j) z_j$$

$$= a \left(\sum_{j=1}^n j^3 x_j z_j \right) + b \left(\sum_{j=1}^n j^3 y_j z_j \right)$$

$$= a \langle x, z \rangle + b \langle y, z \rangle$$

(linearity property)

$$\langle x, x \rangle = \sum_{j=1}^n j^3 x_j^2 \text{ so } \langle x, x \rangle \geq 0 \quad \forall x \in R^n \text{ and}$$

$$\langle x, x \rangle = 0 \Rightarrow \sum_{j=1}^n j^3 x_j^2 = 0$$

$$\Rightarrow x_j = 0; \quad \forall j = 1, 2, \dots, n$$

$$\Rightarrow x = 0$$

Q. 22 Ans (4)

$$T: (p, q) \rightarrow (P, Q)$$

$$Q = pq^{(a+1)} \text{ \& } p = q^b$$

As, $\frac{\partial(P, Q)}{\partial(p, q)} = 1$ (condition for canonical)

$$\Rightarrow \begin{vmatrix} \frac{\partial P}{\partial p} & \frac{\partial P}{\partial q} \\ \frac{\partial Q}{\partial p} & \frac{\partial Q}{\partial q} \end{vmatrix} = 1 \Rightarrow \begin{vmatrix} 0 & bq^{b-1} \\ q^{a+1} & (a+1)pq^a \end{vmatrix} = 1$$

$$\Rightarrow -bq^{a+b} = 1$$

which is satisfied when $a = 1$ $b = -1$

\therefore option (d) is correct.

Q. 23 Ans (3)

$$a_n = (-1)^{n+1} (\sqrt{n+1} - \sqrt{n})$$

$$= (-1)^{n+1} \cdot \frac{(n+1-n)}{\sqrt{n+1} + \sqrt{n}}$$

$$= (-1)^{n+1} \cdot \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$= (-1)^{n+1} \cdot \frac{1}{\sqrt{n}}$$

So, $\sum a_n$ is convergent by

Leibnitz test

but $\sum |a_n| = \sum \frac{1}{\sqrt{n+1} + \sqrt{n}} \approx \sum \frac{1}{\sqrt{n}}$ is divergent series by p - test.

So, $\sum a_n$ is conditionally convergent series.

Q. 24 Ans (4)

Basis of R - vector space of all polynomials in two variables over R having a total degree of at most l will be

$$B = \{1, x, y, x^2, xy, y^2, \dots, x^l, x^{l-1}y, x^{l-2}y^2, \dots, y^l\}$$

so in basis.

Number of vectors of degree 0, $= 1$

Number of vectors of degree 1, $= 2$

Number of vectors of degree 2, $= 3$

\vdots

Number of vector of degree $l = l + 1$

So, dimension of vector space =

Number of vectors in basis $B =$

$$1 + 2 + 3 + \dots + (l+1) = \frac{(l+1)(l+2)}{2}.$$

Q. 25 Ans (4)

$$f(z) = \frac{1}{1-z-z^2} = (1-(z+z^2))^{-1}$$

$$= 1 + (z+z^2) + (z+z^2)^2 + \dots + (z+z^2)^n$$

$$+ (z+z^2)^{n+1} + (z+z^2)^{n+2} + \dots$$

$$a_0 = \text{coefficient of } z^0 = 1$$

$$a_1 = \text{coefficient of } z^1 = 1$$

$$a_n = \text{coefficient of } z^n = 1 + n - 1_{C_1} + n - 2_{C_2}$$

$$a_{n+1} = \text{coefficient of } z^{n+1} = 1 + n_{C_1} + n - 1_{C_2} + \dots$$

$$a_{n+2} = \text{coefficient of } z^{n+2} = 1 + n + 1_{C_1} + n_{C_2} + \dots$$

$$\Rightarrow a_{n+2} = a_n + a_{n+1}$$

$$\therefore n_{C_0} + n_{C_1} = n + 1_{C_1} \Rightarrow 1 + n_{C_1} = n + 1_{C_1} \dots$$

Q. 26 Ans (2)

$$f(x) = x^3 + 3x - 2023$$

$$\Rightarrow f'(x) = 3(x^2 + 1) > 0 ; \forall x \in R$$

$$\Rightarrow f(x) \text{ is strictly increasing in } R$$

So, $f: (-\infty, \infty) \rightarrow (f(-\infty), f(\infty))$ is one-one onto function here $f(-\infty) = -\infty$ & $f(\infty) = \infty$

So, $f: R \rightarrow R$ is bijective map hence $f(x) = 0$ has unique solution.

Q. 27 Ans (4)

Q. 28 Ans (1)

$$y'' + 2y(x) = 0 \text{ for } x \in (0, 1)$$

$$y(0) = y(1) = 0$$

$$y(x) = 2 \int_0^1 k(x, t) y(t) dt \text{ for } x \in [0, 1]$$

so go through option

$$k(x, t) = t(1-x) ; t < x$$

$$= x(1-t) ; t > x$$

$$\therefore y(x) = 2 \int_0^x t(1-x)y dt + 2 \int_x^1 x(1-t)y dt$$

$$y'(x) = 2 \int_0^x -t y(t) dt + x(1-x)y(x).1 - 0$$

$$+ 2 \int_x^1 1(1-t)y(t) dt + 0 - x(1-x)y(x).1$$

$$= 2 \int_0^x -t y(t) dt + 2 \int_x^1 (1-t)y(t) dt$$

$$y'' = 2[0 + (-x y(x)).1 - 0] +$$

$$2[0 + 0 - (1-x)y(x)]$$

$$= -2xy(x) - 2y(x) + 2xy(x)$$

$$= -2y(x)$$

$$y'' = -2y(x)$$

which satisfy the D.E.
Now check boundary condition

$$y(x) = 2 \int_0^x t(1-x)y dt + 2 \int_x^1 x(1-t)y dt$$

$$y(0) = 0 + 0 = 0$$

$$y(1) = 0 + 0 = 0$$

\therefore Option (a) is correct.

Q. 29 Ans (2)

T^2 divides $p(T)$ i.e. characteristic polynomial of A so $p(T) = T^2(T - \alpha)(-1)^3$ so 2 eigenvalues of A must be 0 (real) so third eigen values of A will be also real as non real roots are always in conjugate pairs if coefficient of polynomials are real.

Q. 30 Ans (3)

Q. 31 Ans (4)

$$e^z - e^{-z} = 0 \Rightarrow e^{2z} = 1$$

$$\Rightarrow e^{2z} = e^{i(2k\pi)} ; k \in I \Rightarrow 2z = 2k\pi i$$

$$\Rightarrow z = k\pi i ; k \in I$$

$$\Rightarrow z = 0, \pm \pi i, \pm 2\pi i, \dots$$

Hence, inside $|z - 0| = 3$ we have only one singular point 0 of $\frac{1}{z^2(e^z - e^{-z})}$ which is also pole

of order 3, so $\operatorname{Res}_{z=0} \frac{1}{z^2} (e^z - e^{-z}) =$

$$\begin{aligned} &= \operatorname{Res}_{z=0} \frac{1}{z^2 \left[\left(1 + z + \frac{z^2}{2!} + \dots \right) - \left(1 - z + \frac{z^2}{2!} - \dots \right) \right]} \\ &= \operatorname{Res}_{z=0} \frac{1}{z^2 \cdot \left(2z + \frac{z^3}{3} + \frac{z^5}{60} + \dots \right)} \\ &= \operatorname{Res}_{z=0} \frac{1}{2z^3} \left(1 + \frac{z^2}{6} + \frac{z^4}{120} + \dots \right)^{-1} \\ &= \operatorname{Res}_{z=0} \frac{1 - \left(\frac{z^2}{6} + \frac{z^4}{120} + \dots \right) + \left(\frac{z^2}{6} + \frac{z^4}{120} + \dots \right)^2 + \dots}{2z^3} \\ &= -\frac{1}{12} = \text{coefficient of } \frac{1}{z} \end{aligned}$$

$$\text{So, } \int_C \frac{dz}{z^2 (e^z - e^{-z})} = 2\pi i \left(-\frac{1}{12} \right) = -\frac{\pi i}{6}$$

Q. 32 Ans (3)

Q. 33 Ans (2)

In $(0,1)$

(a) $\because \lim_{x \rightarrow 0^+} \sin \frac{1}{x}$ does not exist

$\therefore \sin \frac{1}{x}$ is not Uniformly Continuous in $(0,1)$

(b) $\lim_{x \rightarrow 0^+} e^{-\frac{1}{x^2}} = 0$ & $\lim_{x \rightarrow 1^-} e^{-\frac{1}{x^2}} = e^{-1} = \frac{1}{e}$

So $e^{-\frac{1}{x^2}}$ is Uniformly Continuous in $(0,1)$

(c) $\because \lim_{x \rightarrow 0^+} e^x \cos \frac{1}{x}$ does not exist. so $e^x \cos \frac{1}{x}$ is not Uniformly Continuous in $(0,1)$

(d) $\because \lim_{x \rightarrow 0^+} \cos x \cos \frac{\pi}{x}$ does not exist so

$\cos x \cos \frac{\pi}{x}$ is not Uniformly Continuous in $(0,1)$

Q. 34 Ans (3)

$$u_{tt} = u_{xx} \quad 0 < x < 2 \quad t > 0$$

$$u(0,t) = 0 = u(2,t)$$

$$u(x,0) = \sin(\pi x) + 2\sin(2\pi x)$$

$$u_t(x,0) = 0$$

Sol:- $c=1 \quad l=2$

Wave equation for finite wave length

$$u(x,t) = \sum_{n=1}^{\infty} \left[a_n \sin\left(\frac{n\pi x}{l}\right) \cos\left(\frac{n\pi t}{l}\right) + b_n \sin\left(\frac{n\pi x}{l}\right) \sin\left(\frac{n\pi t}{l}\right) \right]$$

$$\sin\left(\frac{n\pi x}{l}\right) \sin\left(\frac{n\pi t}{l}\right)$$

$$a_n = \frac{2}{l} \int_0^l f(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$b_n = \frac{2}{n\pi c} \int_0^l g(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$$g(x) = 0 \text{ .Thus } b_n = 0$$

$$f(x) = \sin(\pi x) + 2\sin(2\pi x)$$

$$\text{Now, } \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right) \cos\left(\frac{n\pi t}{2}\right)$$

$$u(x,0) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{l}\right)$$

$$= \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi x}{2}\right)$$

$$= a_1 \sin\left(\frac{\pi x}{2}\right) + a_2 \sin\left(\frac{2\pi x}{2}\right) + a_3 \sin\left(\frac{3\pi x}{2}\right) +$$

$$a_4 \sin(2\pi x)$$

$$\therefore a_4 = 2, a_2 = 1 \quad \therefore a_n = 0 \quad \forall n \neq 2, 4$$

$$u(x,t) = 1 \cdot \sin(\pi x) \cos(\pi t)$$

$$+ 2 \sin(2\pi x) \cos(2\pi t)$$

$$u(1,1) = \sin \pi \cos \pi + 2 \sin(2\pi) \cos(2\pi) = 0$$

$$u\left(\frac{1}{2}, 1\right) = \sin \frac{\pi}{2} \cos \pi + 2 \sin(\pi) \cos(2\pi) = -1$$

$$u\left(\frac{1}{2}, 2\right) = \sin \frac{\pi}{2} \cos 2\pi + 2 \sin \pi \cos 4\pi = 1$$

$$u_t(x, t) = \pi \sin(\pi x) \sin(\pi t) -$$

$$4\pi \sin(2\pi x) \sin(2\pi t)$$

$$u_t\left(\frac{1}{2}, \frac{1}{2}\right) = -\pi \sin \frac{\pi}{2} \sin \frac{\pi}{2} -$$

$$4\pi \sin \pi \sin \pi$$

$$u_t\left(\frac{1}{2}, \frac{1}{2}\right) = -\pi$$

∴ Option (c) is correct.

Q. 35 Ans (1)

Q. 36 Ans (3)

Q. 37 Ans (1)

Q. 38 Ans (1)

Q. 39 Ans (4)

For (a) $T \neq 0$ and 0 is an eigen value of T

$$\Rightarrow f(x) \text{ can be } X(X^2 - 1), \text{ so } g(x) = X^2 - 1$$

$$\Rightarrow g(T) = T^2 - 1 \text{ which need not be } 0$$

So (a) is false

For (b) G.M. of 0 is 2 = A.M. of 0 for T

$$\Rightarrow T \text{ is diagonalisable so } m(x) = X(X - \lambda) \text{ \& }$$

$$f(X) = X^2(X - \lambda)$$

$$\Rightarrow g(x) = X(\lambda - \lambda)$$

$$\Rightarrow g(T) = m(T) = 0$$

If G.M. of 0 is 2 & A.M. of 0 is 3

$$\text{then } f(X) = X^3 \Rightarrow g(X) = X^2 \text{ \& }$$

$$m(X) = X^2$$

$$\Rightarrow g(T) = T^2 = m(T) = 0$$

If G.M. of 0 is 3 then A.M. of 0 is 3

$$\text{then } f(X) = X^3 \text{ and } m(\lambda) = X$$

∴ T is diagonalisable

$$\text{So } g(X) = X^2 \Rightarrow g(T) = T^2 = 0$$

Q. 40 Ans (3)

$$\lim e^{\cos\left(\frac{n\pi(-1)^n \cdot 2e}{2n}\right)} = e^{\cos\left(\frac{\pi}{2}\right)} = e^0 = 1$$

Which is not > 1

$$\lim e^{\log_e\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)} = e^{\log_e\left(\frac{\pi^2}{7}\right)}$$

which exist

$$\lim e^{\sin\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} = e^{\sin\left(\frac{\pi}{2}\right)} = e$$

So

$$\liminf e^{\sin\left(\frac{n\pi + (-1)^n 2e}{2n}\right)} = e$$

Which is less than π . (correct)

$$\lim e^{\tan\left(\frac{n\pi^2 + (-1)^n e^2}{7n}\right)} = e^{\tan\left(\frac{\pi^2}{7}\right)}$$

Which exists.

Q. 41 Ans (1)

$$x, y \in [0, 1] \text{ \& } x \neq y \Rightarrow |x - y| > 0 \text{ \& } \epsilon > 0$$

By Archimedian principle there exist posi-

tive integer N such that $N \cdot \epsilon > |x - y|$

$$\Rightarrow n \in \mathbb{N} \Rightarrow |x - y|; \forall n \geq N$$

$$\therefore 2^n > n \in \mathbb{N} \Rightarrow |x - y|$$

$$\therefore 2^n > |x - y|; \forall n \geq N$$

Q. 42 Ans (2)

Quadratic form associated with matrix

$$B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \text{ is}$$

$$Q(x, y, z) = x^2 + y^2 + 2xy - 2z^2$$

$$\Rightarrow S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in R^3 \mid a^2 + b^2 + 2ab - 2c^2 = 0 \right\}$$

$S \cap X-Y$ plane is $(a+b)^2 = 0 \Rightarrow a+b=0$

$\Rightarrow x+y=0$ which is a line (True)

$S \cap XZ$ plane is $a^2 - 2c^2 = 0$

$\Rightarrow x^2 - 2z^2 = 0$ which is not ellipse (False)

$$S = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in R^3 \mid (a+b)^2 - 2c^2 = 0 \right\}$$

$$= \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in R^3 \mid (a+b+\sqrt{2}c)(a+b-\sqrt{2}c) = 0 \right\}$$

which is union of planes $x+y+\sqrt{2}z=0$ &

$x+y-\sqrt{2}z=0$ (True)

Eigen values of Q are $-2, 2$ & 0

& $\therefore 0$ is an eigenvalue

$\therefore Q$ is degenerate (True)

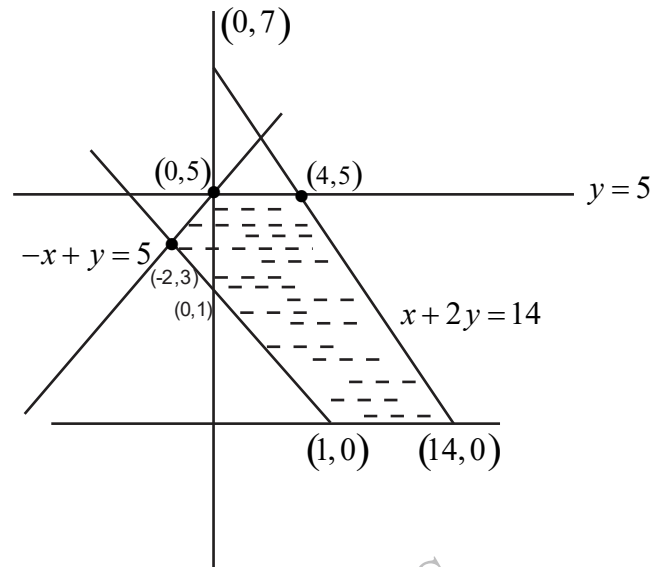
Q. 43 Ans (2)

Q. 44 Ans (4)

Maximize $z = x + 3y$

Subject to

$$\left. \begin{array}{l} -x - y \leq -1 \\ y \leq 5 \\ -x + y \leq 5 \\ x + 2y \leq 14 \\ -y \leq 0 \end{array} \right\} \Rightarrow \begin{array}{l} x + y \geq 1 \\ y \geq 0 \end{array}$$



Vertices of feasible region are $(0,5), (4,5), (-2,3), (1,0)$ & $(14,0)$

$$z(0,5) = 15$$

$$z(4,5) = 19$$

$$z(-2,3) = 7$$

$$z(1,0) = 1$$

$$z(14,0) = 14$$

So, the objective function attains its maximum at $(4,5)$

Q. 45 Ans (3)

$$J(y(x)) = \int_0^1 [y'^2 - y|y|y' + xy] dx$$

Case I:- (for positive $|y|$)

$$y'^2 - y^2 y' + xy$$

$$\Rightarrow \frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$$

$$\Rightarrow -2yy' + x - \frac{d}{dx} (2y'y^2) = 0$$

$$\Rightarrow -2yy' + x - 2y'' + 2yy' = 0$$

$$\Rightarrow y'' = \frac{x}{2}$$

$$\Rightarrow y' = \frac{x^2}{4} + a$$

$$\Rightarrow y(x) = \frac{x^3}{12} + ax + b$$

$$y(0) = 0 \Rightarrow b = 0$$

$$y(1) = 0 \Rightarrow \frac{1}{12} + a = 0$$

$$\Rightarrow a = -\frac{1}{12}$$

$$\therefore y(x) = \frac{x^3}{12} - \frac{1}{12}x = \left(\frac{1}{12}(x^3 - x) \right)$$

There is no Arbitrary Constant, so it give unique solution

\therefore option (c) correct

Case II:- (for negative $|y|$)

$$y'^2 + y^2 y' + xy$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$$

$$2yy' + x - \frac{d}{dx}(2y' + y^2) = 0$$

$$2yy' + x - 2y'' - 2yy' = 0$$

$$\therefore y'' = \frac{x}{2}$$

Q. 46 Ans (1)

$$O(G) = 10$$

(d) $3 \nmid 10$

Thus $1+2+3+4=10$ is not class equation

(c) $10 = 1+1+1+2+5$

$$Z(G) = 3 \quad 3 \nmid 10$$

Thus which is not possible for class equation

(b) $10 = 1+1+2+2+2+2$

$$Z(G) = 2 \quad \left| \frac{G}{Z(G)} \right| \approx \mathbb{Z}_5 \text{ abelian}$$

Then $10 = 1+1+\dots+1$

\therefore (b) is incorrect

\therefore (a) is correct

Q. 47 Ans (3)

$$u u_x + u_y = 0 \quad x \in \mathbb{R}, y > 0$$

$$u(x, 0) = x \quad x \in \mathbb{R}$$

$$P = u \quad Q = 1 \quad R = 0$$

$$\frac{dx}{u} = \frac{dy}{1} = \frac{du}{0}$$

$$\frac{dx}{u} = \frac{du}{0} \Rightarrow u = C_1$$

$$\frac{dx}{C_1} = \frac{dy}{1} \Rightarrow dx = C_1 y$$

$$\Rightarrow x = C_1 y + C_2$$

$$\Rightarrow C_2 = x - uy \quad (2)$$

$$\text{Given } u(x, 0) = x$$

$$\text{Put } x = t \text{ then } u = t \text{ \& } y = 0$$

\therefore From equation (1) and (2)

$$u = C_1 \Rightarrow C_1 = t$$

$$x - uy = C_2 \Rightarrow C_2 = t$$

$$\therefore C_1 = C_2 \Rightarrow u = x - uy$$

$$\Rightarrow u(1+y) = x$$

$$\Rightarrow u = \frac{x}{1+y}$$

$$\therefore u(2, 3) = \frac{2}{1+3} = \frac{1}{2}$$

Option (c) is correct.

Q. 48 Ans (4)

$$A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$$

$$x' = Ax \text{ \& } x(0) = x_0$$

Eigenvalue of A are 1 & 6

$$\therefore x(t) = C_1 e^{6t} + C_2 e^t$$

$$x(0) = C_1 + C_2 = x_0 \Rightarrow C_1 = x_0 - C_2$$

$$\therefore x(t) = (x_0 - C_2) e^{6t} + C_2 e^t$$

$$x(t) = x_0 e^{6t} + C_2 (e^t - e^{6t})$$

(a) As $x_0 \neq 0$ and $t \rightarrow \infty$ then

$x(t) \rightarrow$ unbounded

option (a) is incorrect.

$$(b) \quad e^{-6t} |x(t)| = x_0 + C_2 (e^{-5t} - 1)$$

As $t \rightarrow \infty$

$$e^{-6t} |x(t)| \rightarrow x_0 - C_2$$

option (b) is incorrect

$$(c) \quad e^{-t} |x(t)| = x_0 e^{5t} + C_2 (1 - e^{5t})$$

Let if possible $C_2 = x_0$ then

$$\text{As } t \rightarrow \infty \quad e^{-t} |x(t)| \rightarrow x_0$$

option (c) is incorrect

$$(d) \quad e^{-10t} |x(t)| = x_0 e^{-4t} + C_2 (e^{-9t} - e^{-4t})$$

$$\text{As } t \rightarrow \infty \quad e^{-10t} |x(t)| \rightarrow 0$$

option (d) is correct.

Q. 49 Ans (1)

For degree of precision 3 quadrature formula

will give exact result for $f(x) = 1, x, x^2$ & x^3

$$(i) \quad f(x) = 1$$

$$\int_{-1}^1 1 dx = a + b \Rightarrow a + b = 2 \quad (1)$$

$$(ii) \quad f(x) = x \Rightarrow f'(x) = 1$$

$$\int_{-1}^1 x dx = -a + b + c + d$$

$$\Rightarrow -a + b + c + d = 0 \quad (2)$$

$$(iii) \quad f(x) = x^2 \Rightarrow f'(x) = 2x$$

$$\int_{-1}^1 x^2 dx = a + b - 2c + 2d$$

$$\Rightarrow a + b - 2c + 2d = \frac{2}{3} \quad (3)$$

$$(iv) \quad f(x) = x^3 \Rightarrow f'(x) = 3x^2$$

$$\int_{-1}^1 f(x) dx = -a + b + 3c + 3d$$

$$\Rightarrow -a + b + 3c + 3d = 0 \quad (4)$$

(1), (2), (3) & (4)

$$a = 1; b = 1; c = \frac{1}{3} \text{ \& } d = -\frac{1}{3}$$

Q. 50 Ans (3)

If S is an infinite set then

$$(i) \quad \text{If } |S| = \aleph_0 \text{ then}$$

$$S \sim N \text{ \& } \because N \times N \sim N \therefore S \times S \sim S$$

$$(ii) \quad \text{If } |S| = c \text{ then}$$

$$S \sim R \because R \times R \sim R \therefore S \times S \sim S$$

So there is a bijection of S with $S \times S$

Q. 51 Ans (4)

Q. 52 Ans (4)

$$\frac{\mathbb{Z}}{105\mathbb{Z}} \approx \mathbb{Z}_{105}$$

No. of solution of equation $x^2 = 1$ is equal to

number of element of order 1 & 2 in $U(105)$

$$U(105) = U(3) \times U(5) \times U(7)$$

$$= \mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$$

Thus No. of solution = 8

Option (d) is correct

Q. 53 Ans (3)

$$f(z) = e^{z + \frac{1}{z}} = e^z e^{\frac{1}{z}}$$

$$= \left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots \right) \left(1 + \frac{1}{z} + \frac{1}{z^2 2!} + \dots \right)$$

So, coefficient of $\frac{1}{z}$ in

$$f(z) = \frac{1}{0!1!} + \frac{1}{1!2!} + \frac{1}{2!3!} + \dots = \sum_{i=0}^{\infty} \frac{1}{i!(i+1)!}$$

$$\Rightarrow \operatorname{Res}_{z=0} f(z) = \sum_{i=0}^{\infty} \frac{1}{i!(i+1)!}$$

Q. 54 Ans (4)

$$(a) \quad |G| = P^6 \text{ (Given)}$$

Let $H \leq G$ and $O(H) = P^4$ then

$$[G:H] = \frac{O(G)}{O(H)} = P^2$$

which is true statement

(c) $G = \mathbb{Z}_p \times \mathbb{Z}_p \times \dots \times \mathbb{Z}_p$ (infinite time)

$$Z(G) = \infty$$

True statement

If $O(G) = P^n$ then \exists at least one normal

subgroup of order $P^r, \forall 0 \leq r \leq n$

\therefore (b) is True statement

Option (d) is correct.

Q. 55 Ans (2)

Q. 56 Ans (1)

Function

$$g(z) \text{ such that } |g(z)| = e^y \text{ is } g(z) = e^{-iz}.$$

Now if $|f(z)| \leq e^y \quad \forall z \in C$ then

$$|f(z)| \leq |g(z)| \quad \forall z \in C$$

$$\Rightarrow f(z) = c g(z); \forall z \in C \text{ where } |c| \leq 1 \text{ so}$$

$$f(z) = c e^{-iz}; \forall z \in C \text{ with } |c| \leq 1$$

Q. 57 Ans (2)

Q. 58 Ans (2)

Q. 59 Ans (4)

If order of A is odd and A is real matrix then its characteristic polynomial is of odd degree with real coefficient so it must have at least one real root. Hence as order (A) is 3 (odd) so it must have at least one real eigen value. (1) is true.

$\therefore |A|$ is product of eigen values of A so

$$|A| = 0 \Rightarrow A \text{ has at least one eigen value } 0.$$

(2) is correct.

$|A| < 0$ i.e. product of 3 eigen values of A is negative and if one eigen value of A is 3 then product of remaining 2 eigen values of A is negative so remaining two eigen values cannot be real because product of non real eigen values are positive as

$(\alpha + i\beta)(\alpha - i\beta) = \alpha^2 + \beta^2$ so all 3 eigen values of A are real

(3) is correct.

Now if $|A| > 0$ and 3 is an eigen value of A then remaining 2 eigen values of A can be

$$i \& -i \text{ e.g., see } A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

So, option (4) is false.

Q. 60 Ans (3)

PART "C"

Q. 61 Ans (2,4)

$f: R^n \rightarrow R; n \geq 2$ is convex iff

$$f(\lambda y + (1-\lambda)x) \leq \lambda f(y) + (1-\lambda)f(x)$$

$$\forall \lambda \in [0,1]; x, y \in R^n$$

$$\Rightarrow f(x + \lambda(y-x)) \leq \lambda(f(y) - f(x)) + f(x)$$

$$\Rightarrow \lambda(f(y) - f(x)) \geq f(x + \lambda(y-x)) - f(x)$$

$$\Rightarrow f(y) - f(x) \geq \frac{f(x + \lambda(y-x)) - f(x)}{\lambda}$$

$$\Rightarrow f(y) - f(x) \geq (y-x) \cdot \left[\frac{f(x + \lambda(y-x)) - f(x)}{\lambda(y-x)} \right] \quad (1)$$

In (1) take $\lambda \rightarrow 0^+$

$$\Rightarrow f(y) - f(x) \geq (\nabla(f)(x))(y-x)$$

$$\Rightarrow f(y) \geq f(x) + (\nabla(f)(x))(y-x)$$

Option (2)

Now if f is bounded then $\forall x, y$,

$f(y) - f(x)$ is bounded, so take

$$y = x + \vec{n}; n \in N; \forall x \in R^n$$

$$f(y) \geq f(x) + \nabla(f)(x)(y-x)$$

$$\Rightarrow \frac{f(y) - f(x)}{y-x} \geq \nabla(f)(x)$$

$$\Rightarrow \forall x \in R^n, \nabla(f)(x) \leq \lim_{n \rightarrow \infty} \frac{f(x + \vec{n}) - f(x)}{\vec{n}}$$

$$\Rightarrow \nabla(f)(x) \leq 0 \Rightarrow \nabla(f)(x) = 0$$

$$\Rightarrow f(x) \text{ is constant.}$$

Q. 62 Ans (1,2,4)

Q. 63 Ans (1,3,4)

Q. 64 Ans (1,2,3)

McLaurin's expansion of $f(z)$ about 0 is

$$f(z) = f(0) + f'(0)z + f''(0)\frac{z^2}{2!} + \dots$$

$$\Rightarrow f(z) = z f'(0) + \frac{z^2}{2!} f''(0) + \dots$$

Let ,
$$g(z) = \frac{f(z)}{z} ; z \neq 0$$

$$= f'(0) ; z = 0$$

where $g(z)$ is analytic in $|z| < 1$ so,

$$|g(z)| = \left| \frac{f(z)}{z} \right| \leq \frac{1}{2|z|} \leq \frac{1}{2} , \text{ when } |z| \rightarrow 1$$

$$\Rightarrow |f(z)| \leq \frac{1}{2}|z| ; \forall z : |z| < 1$$

$$\Rightarrow |f(z)| \leq |z| ; \forall z : |z| < 1$$

$$\Rightarrow |f(z)| \leq \frac{1}{2} ; \forall z : |z| < 1$$

so, $f\left(\frac{1}{2}\right) = \frac{1}{2}$ is not possible else $f(z)$ will be constant.

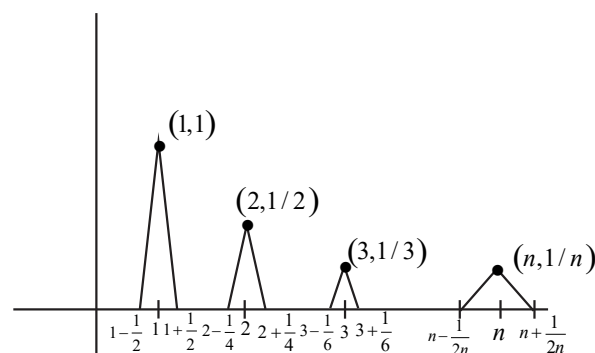
Q. 65 Ans (1,2,3)

; Q. 66 Ans (2,4)

Q. 67 Ans (1,3)

Q. 68 Ans (2,3)

(a) is False



$$\Rightarrow \int_1^{\infty} f(x) dx \leq \frac{1}{2} \left(1^2 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2} + \dots \right) = \text{finite but}$$

$$\sum f(n) = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} + \dots = \infty$$

(b) $f(x) = 1 ; x \in N$
 $= 0 ; x \notin N$

$$\Rightarrow \lim_{x \rightarrow \infty} f(x) \text{ does not exist and } \int_1^{\infty} f(x) dx = 0$$

so, (b) is also false

(c) By Cauchy's integral test $\int_1^{\infty} f(x) dx$ and

$$\sum_{n=1}^{\infty} f(n) \text{ both converges or diverges together.}$$

So, (c) is true

Q. 69 Ans (2,3)

Q. 70 Ans (1,3)

Q. 71 Ans (1,2,3)

Q. 72 Ans (2)

(1) $f_n(x) = x^n ; x \in (0,1)$

$$\Rightarrow f(x) = \lim_{n \rightarrow \infty} f_n(x) = \lim_{n \rightarrow \infty} x^n = 0 ; \forall x \in (0,1)$$

$$\sup_{x \in (0,1)} |f_n(x) - f(x)| = \sup_{x \in (0,1)} |x^n| = 1 = M_n$$

$$\lim_{n \rightarrow \infty} M_n = \lim_{n \rightarrow \infty} 1 = 1 \neq 0$$

$$\Rightarrow \{f_n(x)\} \text{ do not converge uniformly in } (0,1)$$

(2) $f_n(x) = \frac{x^n}{\log(n+1)}$

$$\Rightarrow f(x) = \lim_{n \rightarrow \infty} \frac{x^n}{\log(n+1)} = 0 ; \forall x \in (0,1)$$

hence ,

$$|f_n(x) - f(x)| = \frac{x^n}{\log(n+1)} ; \forall x \in (0,1)$$

$$\Rightarrow M_n = \sup_{x \in (0,1)} |f_n(x) - f(x)|$$

$$= \sup_{x \in (0,1)} \frac{x^n}{\log(n+1)} = \frac{1}{\log(n+1)}$$

$$\text{Hence } \lim_{n \rightarrow \infty} M_n = \lim_{n \rightarrow \infty} \frac{1}{\log(n+1)} = 0$$

So, $\{f_n(x)\}$ converges uniformly to 0 on $(0,1)$

$$(3) \quad f_n(x) = \frac{x^n}{1+x^n}$$

$$\Rightarrow f(x) = \lim_{n \rightarrow \infty} f_n(x) = \lim_{n \rightarrow \infty} \frac{x^n}{1+x^n} = 0 \quad ; \quad x \in (0,1)$$

$$|f_n(x) - f(x)| = \frac{x^n}{1+x^n} \quad ; \quad \forall x \in (0,1)$$

$$M_n = \sup_{x \in (0,1)} |f_n(x) - f(x)| \geq \left. \frac{x^n}{1+x^n} \right|_{x=1} = \frac{1}{2}$$

$$\Rightarrow \lim M_n \geq \frac{1}{2} \neq 0$$

So convergence is not uniform

$$(4) \quad f_n(x) = \frac{x^n}{1+nx^n}$$

$$\Rightarrow f(x) = 0 \quad ; \quad \forall x \in (0,1)$$

$$|f_n(x) - f(x)| = \frac{x^n}{1+nx^n}$$

$$\text{so, } \forall x \in [0,1] \quad \sup |f_n(x) - f(x)| = M_n \rightarrow 0 \text{ as } n \rightarrow \infty$$

Q. 73 Ans (1,3)

$$J(y(x)) = \int_0^1 (y(x)^2 - 4y(x)y'(x) +$$

$$4y'(x)^2) dx$$

$$f = y^2 - 4yy' + 4y'^2$$

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$$

$$\Rightarrow 2y - 4y' - \frac{d}{dx}(-4y + 8y') = 0$$

$$\Rightarrow 2y - 4y' + 4y' - 8y'' = 0$$

$$\Rightarrow 4y'' - y = 0 \quad \Rightarrow 4M^2 - 1 = 0$$

$$\Rightarrow M = \pm \frac{1}{2}$$

$$\therefore y(x) = C_1 e^{\frac{1}{2}x} + C_2 e^{-\frac{1}{2}x}$$

$$y(0) = 1 \quad \Rightarrow C_1 + C_2 = 1$$

$$y'(0) = \frac{1}{2} \quad \Rightarrow \frac{C_1}{2} - \frac{C_2}{2} = \frac{1}{2} = C_1 - C_2 = 1$$

$$\therefore C_1 = 1 \quad C_2 = 0$$

$$\therefore y(x) = e^{\frac{1}{2}x}$$

$$y' = \frac{1}{2} e^{\frac{1}{2}x}$$

$$y''(x) = \frac{1}{4} e^{\frac{1}{2}x} > 0 \quad ; \quad \forall x$$

$\therefore y$ is convex

$$y(x_1 + x_2) = e^{\frac{x_1 + x_2}{2}}$$

$$= e^{\frac{x_1}{2}} e^{\frac{x_2}{2}}$$

$$= y(x_1) y(x_2)$$

\therefore Option (a) & (c) is correct.

Q. 74 Ans (3)

$$f(z) = u + iv \text{ is entire function, so } u_x = v_y$$

$$\& u_y = -v_x \quad (1)$$

Also u_x, u_y, v_x & v_y are continuous in R^2

$$\text{Now } f(z) = f(x + iy) = u(x, y) + iv(x, y)$$

$$\Rightarrow f(\bar{z}) = f(x - iy) = u(x, -y) + iv(x, -y)$$

$$\text{And } \overline{f(\bar{z})} = \overline{f(x - iy)} = g(z) =$$

$$u(x, -y) - iv(x, -y)$$

Real part and imaginary part of $g(z)$ are

$$u_1 = u(x, -y) \text{ \& } v_1 = -v(x, -y)$$

$$(u_1)_x = u_x(x, -y); (v_1)_y = +v_y(x, -y) =$$

$$v_y(x, -y)$$

$$(u_1)_y = -u_y(x, -y); v_1(x) = -v_x(x, -y)$$

$$\text{From (1) } (u_1)_x = (v_1)_y$$

$$\text{\& } (u_1)_y = -(v_1)_x$$

So, $g(z) = \overline{f(\bar{z})}$ is analytic

Option (3) is correct.

$$f(z) = z \Rightarrow f(\bar{z}) = \bar{z} \text{ here } f(z) \text{ is entire}$$

but $f(\bar{z})$ is not entire

$f(z) = z \Rightarrow \overline{f(z)} = \bar{z}$ so as above $\overline{f(z)}$ is not entire.

Now $\overline{f(z)} + f(\bar{z}) = 2\bar{z}$ for $f(z) = z$ so again it is not entire.
(Options 1, 2, & 4 are wrong)

Q. 75 Ans (1,3)

$$\phi(x) = \lambda \int_0^{2\pi} \sin(x+t) \phi(t) dt$$

$$k(x, t) = \sin(x+t) = \sin x \cos t + \cos x \sin t$$

$$\text{Let } f_1 = \sin x \quad f_2 = \cos x$$

$$g_1 = \cos t \quad g_2 = \sin t$$

$$\alpha_{ij} = \int_a^b g_i(t) f_j(t) dt$$

$$\beta = \int_a^b g_i(t) f(t) dt$$

$$\alpha_{11} = \int_0^{2\pi} \sin x \cos x ds = \int_0^{2\pi} \frac{\sin 2x}{2} dx = 0$$

$$\alpha_{12} = \int_0^{2\pi} \cos^2 x dx = \pi$$

$$\alpha_{21} = \int_0^{2\pi} \sin^2 x dx = \pi$$

$$\alpha_{22} = \int_0^{2\pi} \frac{\sin 2x}{2} dx = 0$$

$$A = \begin{pmatrix} 1 - \lambda \alpha_{11} & -\lambda \alpha_{12} \\ -\lambda \alpha_{21} & 1 - \lambda \alpha_{22} \end{pmatrix} = \begin{pmatrix} 1 - \lambda \cdot 0 & -\lambda \pi \\ -\lambda \pi & 1 - \lambda \cdot 0 \end{pmatrix}$$

$$|A| = 0 \Rightarrow |A| = 1 - \lambda^2 \pi^2 = 0$$

$$\therefore \lambda = \pm \frac{1}{\pi}$$

$$\therefore \lambda_1 = \frac{-1}{\pi} \text{ \& } \lambda_2 = \frac{1}{\pi}$$

$$\text{Now } \psi(x) = u \int_0^\pi \cos(x+t) \psi(t) dt$$

$$k(x, t) = \cos(x+t) = \cos x \cos t - \sin x \sin t$$

$$f_1 = \cos x \quad g_1 = \cos t$$

$$f_2 = -\sin x \quad g_2 = \sin t$$

$$\alpha_{11} = \int_0^\pi \cos^2 x = \frac{\pi}{2} \quad \alpha_{12} = \int_0^\pi \frac{-\sin 2x}{2} = 0$$

$$\alpha_{21} = \int_0^\pi \frac{\sin 2x}{2} = 0 \quad \alpha_{22} = \int_0^\pi -\sin^2 dx = \frac{-\pi}{2}$$

$$A = \begin{pmatrix} 1 - \mu \alpha_{11} & -\mu \alpha_{12} \\ -\mu \alpha_{21} & 1 - \mu \alpha_{22} \end{pmatrix} = \begin{pmatrix} 1 - \mu \frac{\pi}{2} & -\mu \cdot 0 \\ -\mu \cdot 0 & 1 + \mu \frac{\pi}{2} \end{pmatrix}$$

$$|A| = 1 - \mu^2 \frac{\pi^2}{4} = 0 \Rightarrow \mu = \pm \frac{2}{\pi}$$

$$\mu_1 = \frac{-2}{\pi} \quad \mu_2 = \frac{2}{\pi}$$

$\therefore \mu_2 > \lambda_2 > \lambda_1 > \mu_1$, option (a) (c) is correct

Q. 76 Ans (1,2)

If $\{x_{2n}\}$ & $\{x_{2n+1}\}$ are convergent and have same limit then $\{x_n\}$ is convergent as natural numbers are either even or odd and both sequence of odd & even terms have same limit so $\{s_n\}$ has unique limit point.

Further if $\{x_{2n}\}$, $\{x_{2n+1}\}$ and $\{x_{3n}\}$ converges to l, m & p respectively then $\{x_{6n}\}$ is sub

sequence of both $\{x_{2n}\}$ & $\{x_{3n}\}$ and hence it will converge to l & p so $l = p$.

Also $\{x_{3(2n+1)}\}$ is subsequence of both $\{x_{3n}\}$ & $\{x_{2n+1}\}$ so it converges to m & p both so $m = p$ and hence $l = m$ so both $\{x_{2n}\}$ & $\{x_{2n+1}\}$ converges to same limit so $\{x_n\}$ is convergent.

(Option 1 & 2 are correct)

Take sequence $\{x_n\}$ where

$$x_n = 10 \text{ when } n \text{ is prime number} \\ = 9 \text{ when } n \text{ is not prime number}$$

Now every subsequence $\{x_{k_n}\}$ for every $k \geq 2$ is convergent and converges to 9 as except possibly first term all terms of subsequence are 9 i.e. it is ultimately constant sequence $\{9\}$ but sequence $\{x_n\}$ is not convergent, it oscillates between 9 & 10.

If we take $x_n = \sqrt{n}$ then

$$\lim_{n \rightarrow \infty} |x_{n+1} - x_n| = \lim_{n \rightarrow \infty} |\sqrt{n+1} - \sqrt{n}| = 0 \quad \text{but}$$

$$\{x_n\} = \{\sqrt{n}\} \text{ is not convergent.}$$

Q. 77 Ans (1,4)

Q. 78 Ans (1,3)

$$J(y(x), z(x)) = \int_0^1 (y'^2 + z'^2 + y'z') dx$$

$$y(0) = 1 \quad y(1) = 0$$

$$z(0) = -1 \quad z(1) = 2$$

Given problem is independent of x, y and x, z
∴ Solution is

$$y = ax + b \quad z = cx + d$$

$$y(0) = 1 \Rightarrow b = 1 \quad z(0) = -1$$

$$y(1) = 0 \Rightarrow a = -1 \quad \Rightarrow d = -1$$

$$\therefore y(x) = -x + 1 \quad z(1) = 2$$

$$\Rightarrow c - 1 = 2 \Rightarrow c = 3$$

$$\therefore z = 3x - 1$$

$$\therefore y + z = 2x \quad \text{option (c) is correct}$$

$$z + 3y = 3x - 1 - 3x + 3 = 2$$

option (a) is correct

Q. 79 Ans (1,3)

Q. 80 Ans (1,4)

$$O(G) = 2023 = 7 \times 17 \times 17$$

$$|G| = p^2 q \quad p \nmid q-1 \quad q \nmid p^2-1$$

Then G is abelian

option (a) is correct.

$$2023 = 7 \times 17^2$$

$7^1 \mid 2023$ but $7^{1+1} \nmid 2023$ then any subgroup of order 7 is 7-SSG

$17^2 \mid 2023$ but $17^{2+1} \nmid 2023$ then any subgroup of order 289 is 17-SSG

$$\text{No. of 7-SSG} = n_7 = 1 + 7k$$

$$1 + 7k \mid 289$$

$$\Rightarrow k = 0 \text{ then } n_7 = 1 \text{ unique 7-SSG is normal}$$

$$\text{No. of 17-SSG } n_{17} = 1 + 17k$$

$$1 + 17k \mid 7$$

$$\therefore k = 0 \text{ then } n_{17} = 1 \text{ unique 17-SSG is normal}$$

$$\therefore G \approx \mathbb{Z}_7 \times \mathbb{Z}_{17} \times \mathbb{Z}_{17}$$

$$\therefore G \text{ is abelian but not cyclic.}$$

Since 7-SSG and 17-SSG is normal thus G is not a simple group.
option (a) & (d) is correct.

Q. 81 Ans (2)

Q. 82 Ans (1,2,4)

$$(P) = \begin{cases} x'(t) = f(x(t)) & t > 0 \\ x(0) = 0 \end{cases}$$

$$C'(R) \rightarrow \text{It's diff. + bounded}$$

$$\text{Let } f' = \text{Constant} = 1 \text{ (say)}$$

$$\therefore x' = 1 \Rightarrow x = t \text{ (unique solution)}$$

$$\forall t > 0$$

$$\therefore \text{option (a) (b) \& (d) is correct.}$$

Q. 83 Ans (1)

Q. 84 Ans (1,2,3,4)

$\phi: G_1 \longrightarrow G_2$ onto group homomorphism

By properties of homomorphism.

∴ All options are correct.

Q. 85 Ans (1,3)

$B \in M_{3 \times 5}(Q)$ & $W = \{v \in R^5 \mid BV = 0\}$ is a three dimensional real vector space so $\dim W = 3 = 5 - \text{rank}(B) \Rightarrow \text{Rank}(B) = 2$

hence $W_1 = \{V \in Q^5 \mid BV = 0\}$ over Q has dimension $= 5 - \text{Rank}(B) = 5 - 2 = 3$

(option 1 is correct)

Linear transformation $T: Q^3 \rightarrow Q^5$ given by

$T(V) = B^tV$ has Null space as

$N(T) = \{V \in Q^3 \mid T(V) = B^tV = 0\}$ and

$\dim(N(T)) = \dim(Q^3) - \text{Rank}(B^t)$

$= 3 - 2 = 1 \neq 0$

So, T is not injective.

(option 2 is incorrect)

As $\text{Rank}(B) = 2$ so B has 2 linearly independent column vectors.

So, column span of B is two dimensional vector space.

(option 3 is correct)

∴ B is real matrix

∴ $\text{Rank}(BB^t) = \text{Rank}(B) = 2$ so Linear trans-

formation $T: Q^3 \rightarrow Q^5$ given by

$T(V) = BB^tV$ has nullity 1 and rank 2. so it is not injective. (option 4 is not correct)

Q. 86 Ans (1,2,3)

$T^3 - 3T^2 = -2I \Rightarrow T^3 - 3T^2 + 2I = 0$

$\Rightarrow (T - I)(T^2 - 2T - 2I) = 0$

$\Rightarrow (T - I)((T - I)^2 - (\sqrt{3}I)^2) = 0$

$\Rightarrow (T - I)(T - (\sqrt{3} + 1)I)(T - (1 - \sqrt{3})I) = 0$

So an annihilating polynomial of T will be

$(x - 1)(x - (1 + \sqrt{3}))(x - (1 - \sqrt{3}))$ hence

minimal polynomial of T has every factor in linear form so T is diagonalisable, hence there exist a basis B_1 of R^3 w.r.t. which matrix of T is diagonal which is both symmetric and upper triangular.

(option 1 & 2 are correct)

Now if $T = (1 - \sqrt{3})I$ or $T = (1 + \sqrt{3})I$ then option 3 & option 4 both are incorrect.

Q. 87 Ans (2,3,4)

$\forall C \in (0, 1)$ as C is an interior point of $(0, 1)$

so $\lim_{x \rightarrow C} f(x) = \lim_{x \rightarrow C} \sin\left(\frac{\pi}{x}\right) = \sin\left(\frac{\pi}{C}\right)$

$= f(C)$. So, $f(x)$ is continuous in $(0, 1)$ option 2 is correct.

In set of points, $S = \{0\} \cup \left\{\frac{1}{n} \mid n \in N\right\}$

$f(x) = 0; \forall x \in S$,

so $f(x)$ is continuous in S .

option 3 is correct.

For set of points, $S = \left\{\frac{1}{2^n}; n \in N\right\}$ each point

of S is isolated point, so $f(x)$ is continuous in S .

option 4 is correct.

For $S = [-1, 1]$, $0 \in [-1, 1]$ and 0 is interior point of S but $\lim_{x \rightarrow 0} f(x)$ does not exist, so

$f(x)$ is not continuous in $[-1, 1]$.

option 1 is wrong.

Q. 88 Ans (3,4)

Q. 89 Ans (1,2,3,4)

∴ $\inf(A) \subseteq A$ & $A \subseteq \overline{A}$ and

$\mu^*(\inf A) = \mu^*(\overline{A})$ so $\mu_*(A) = \mu^*(A)$ so A is Lebesgue measurable.

(option 1)

$A \neq \phi$ & $A \subseteq [0, 1]$ and A is open set so A contains a non-trivial interval hence $\mu(A) > 0$

and it is measurable as it is union of open intervals.
(option 2)

As measure of rationals is zero (0) so A is not measurable implies that the set of irrationals in A must have positive outer measure.
(option 3)

It $m^*(A) = 0$ then A cannot contain a non-trivial interval so A has empty interior.
(option 4)

Q. 90 Ans (2,4)

$$\frac{dx}{1} = \frac{dy}{1} = \frac{du}{e^u}$$

$$dx = dy \Rightarrow x - y = C_1 \quad (1)$$

$$\frac{dy}{1} = \frac{du}{e^u} \Rightarrow y + e^{-u} = C_2 \quad (2)$$

$$\phi(x - y) = y + e^{-u}$$

$$u(x, 0) = 1$$

$$\therefore y = 0 \quad x = t \quad u = 1$$

$$\phi(x) = e^{-1} = \frac{1}{e}$$

$$\frac{1}{e} = y + e^{-u}$$

$$\Rightarrow u = -\log\left(\frac{1}{e} - y\right)$$

$$(a) \quad u\left(\frac{1}{2e}, \frac{1}{2e}\right) = -\log\left(\frac{1}{e} - \frac{1}{2e}\right) = -\log\left(\frac{1}{2e}\right) \neq 1$$

$$u_x = 0 \text{ and } u_y = \frac{e}{1 - e^y}$$

$$\therefore u_y\left(\frac{1}{4e}, \frac{1}{4e}\right) = \frac{e}{1 - \frac{1}{4e}} = \frac{4e}{3}$$

\therefore Option (b) & (d) is correct.

Q. 91 Ans (2,4)

$$V = P_{10}(R) \Rightarrow \dim V = 10 + 1 = 11 \quad (1)$$

$$W_1 = P_5(R) \Rightarrow \dim W_1 = 5 + 1 = 6 \quad (2)$$

$$W_2 = \{a_0 + a_1x + a_2x^2 + \dots + a_{10}x^{10} \mid$$

$$a_i \in R; i = 0, 1, 2, \dots, 10 \text{ \& } a_0 + a_1 + \dots + a_{10} = 0\}$$

$$\Rightarrow \dim W_2 = 11 - 1 = 10 \quad (3)$$

$$W = L(W_1 \cup W_2) = W_1 + W_2 \quad (4)$$

$$W_1 \cap W_2 = \{a_0 + a_1x + \dots + a_5x^5 \mid$$

$$a_0 + a_1 + \dots + a_5 = 0\}$$

$$\Rightarrow \dim(W_1 \cap W_2) = 6 - 1 = 5 \quad (5)$$

option (4)

$$\therefore \dim(W) = \dim(W_1 + W_2) =$$

$$\dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2) =$$

$$6 + 10 - 5 = 11 = \dim V$$

So, $W = V$

option (2)

Q. 92 Ans (1,2,3,4)

$$V = P_{10}(R);$$

$T: V \rightarrow V$ is given by $T(p) = p'$ so matrix of T will be

$$T = \begin{matrix} & T(1) & T(x) & T(x^2) & T(x^{10}) \\ \begin{matrix} 1 \\ x \\ x^2 \\ \vdots \\ x^{10} \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & \vdots \\ 0 & 0 & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}_{11 \times 11}$$

\Rightarrow First 10 rows of T are non-zero row and in Echelon form type, so

$$(i) \quad \text{Rank}(T) = 10$$

$$(ii) \quad |T| = 0$$

$$(iii) \quad \text{trace}(T) = \sum_{i=1}^{11} t_{ii} = 0$$

(iv) All eigenvalues of T are 0

Q. 93 Ans (1,3)

$$f(x) = \frac{1}{4} + x - x^2 = \frac{1}{4} + \frac{1}{4} - \left(x^2 - x + \frac{1}{4}\right)$$

$$= \frac{1}{2} - \left(x - \frac{1}{2}\right)^2 \quad (1)$$

So, $x_{n+1} = f(x_n)$

$$\Rightarrow x_{n+1} = \frac{1}{2} - \left(x_n - \frac{1}{2}\right)^2$$

$$\Rightarrow \left(\frac{1}{2} - x_{n+1}\right) = \left(\frac{1}{2} - x_n\right)^2 \quad (2)$$

Let $\left|x_n - \frac{1}{2}\right| = \epsilon_n$ (error in nth iteration)

$$\Rightarrow \epsilon_{n+1} = \epsilon_n^2$$

$$\Rightarrow \epsilon_{n+1} = (\epsilon_{n-1})^2 \dots$$

$$\Rightarrow \epsilon_{n+1} = (\epsilon_0)^{2^{n+1}}$$

If $|\epsilon_0| < 1$ then $\lim_{n \rightarrow \infty} \epsilon_{n+1} = 0$

So $\{x_n\}$ converges to $\frac{1}{2}$ if $\left|x_0 - \frac{1}{2}\right| < 1$

$$\Rightarrow x_0 = a \in \left(\frac{1}{2} - 1, \frac{1}{2} + 1\right) \text{ i.e. } a \in \left(-\frac{1}{2}, \frac{3}{2}\right)$$

Q. 94 Ans (1,2,3,4)

Q. 95 Ans (1,3)

$$\frac{du}{dt} = t^2 u^{1/5} \quad u(0) = 0$$

$$f(u, t) = t^2 u^{1/5}$$

$$\frac{du}{dt} = Au^\alpha \quad y(\beta) = 0 \text{ has}$$

i) Unique solution if $A < 0$; $\alpha \in (0, 1)$ & $\beta \in \mathbb{R}$

ii) Infinite solution if $A > 0$; $\alpha \in (0, 1)$ & $\beta \in \mathbb{R}$

$$t^2 > 0$$

This IVP has infinite solution
option (c) is correct

$$\frac{\partial f}{\partial u} = \frac{1}{5} t^2 u^{-4/5}$$

$\therefore \frac{\partial f}{\partial u}$ does not exist at $u = 0$

\therefore Lipschitz condition is not satisfied
 \therefore option (a) is correct

Q. 96 Ans (2)

$f: S_n \times S_n \rightarrow S_n$ defined by $f(a, b) = ab$

Let (x_1, y_1) and $(x_2, y_2) \in S_n \times S_n$

$$f((x_1, y_1) * (x_2, y_2)) = f((x_1 x_2, y_1 y_2)) \\ = x_1 x_2 y_1 y_2$$

$$\neq (x_1 y_1)(x_2 y_2) \quad \forall (x_1, y_1) \& (x_2, y_2) \in S_n \times S_n$$

($\because S_n$ $n \geq 3$ non commutative)

Option (a) is wrong

$$(b) \quad \Delta = \{(g, g) \mid g \in S_n\}$$

$$e \in S_n \Rightarrow (e, e) \in \Delta \Rightarrow \Delta \neq \emptyset$$

$$\forall (x, x) \& (y, y) \in \Delta$$

$$(x, x) * (y, y)^{-1} = (x, x) * (y^{-1}, y^{-1})$$

$$= (x y^{-1}, x y^{-1}) \in \Delta$$

$$\therefore \Delta \leq S_n \times S_n$$

Option (b) is correct.

(c) $G \rightarrow$ Group & $H \leq G$

Let $g = (g_1, g_2) \in S_n \times S_n$ and $x = (x_1, x_1) \in \Delta$

$$\therefore g^{-1} x g = (g_1, g_2)^{-1} * (x_1, x_1) * (g_1, g_2)$$

$$= (g_1^{-1} x_1 g_2, g_1^{-1} x_1 g_2) \notin \Delta \quad \forall g_1, g_2 \in S_n$$

$\therefore \Delta$ is not normal in $S_n \times S_n$

\therefore option (c) is wrong

Q. 97 Ans (2,4)

Q. 98 Ans (2,4)

Q. 99 Ans (2,3)

$$|K(x, y)| < 1$$

$\therefore K(x, y)$ is bounded $\forall x, y \in [0, 1]$

Here $\lambda = 1$

$$A = \begin{pmatrix} 1 - \lambda \alpha_{11} & -\lambda \alpha_{12} \\ -\lambda \alpha_{21} & 1 - \lambda \alpha_{22} \end{pmatrix} = \begin{pmatrix} 1 - \alpha_{11} & -\alpha_{12} \\ -\alpha_{21} & 1 - \alpha_{22} \end{pmatrix}$$

As $|K(x, y)| < 1$ so $\alpha_{11} \neq 0$ $\alpha_{22} \neq 0$

$\therefore |A| \neq 0$ $\rho(A) = 2$

\therefore Option (b) & (c) are correct

$$K(x, y) = \frac{x}{2}; f = \frac{x}{2} \text{ \& } g = 1$$

$$\text{Let } A = (1 - \lambda \alpha_{11}); \alpha_{11} = \int_0^1 \frac{x}{2} dx = \frac{1}{4}$$

$$|A| \neq 0 \therefore \rho(A) = n$$

\therefore Option (a) & (d) are wrong

Q. 100 Ans (1,2,3,4)

$f: R^4 \rightarrow R$ is function given by

$$f(x, y, z, w) = xw - yz \text{ then}$$

(1) f is continuous as it is polynomial in x, y, z & w

(2) In $U = \{(x, y, z, w) \in R^4; xy + zw = 0,$

$$x^2 + z^2 = 1, y^2 + w^2 = 1\}$$

$$x^2 + z^2 = 1 \Rightarrow |x| \leq 1 \text{ \& } |z| \leq 1$$

$$y^2 + w^2 = 1 \Rightarrow |y| \leq 1 \text{ \& } |w| \leq 1$$

So U is contained in a compact region and f is continuous there, so it is uniformly continuous in U

(3) $V = \{(x, y, z, w) \in R^4; x = y, z = w\}$

\Rightarrow In $V, f(x, y, z, w) = xw - yz$

$= xz - xz = 0$ which is constant so it is uniformly continuous in V

(4) In

$$W = \{(x, y, z, w) \in R^4; 0 \leq x + y + z + w \leq 1\}$$

$w = -x$ & $z = -y$ satisfy constraint in W but at $w = -x$ & $z = -y$

$$f(x, y, z, w) = xw - yz = -x^2 + y^2 \text{ and}$$

$$f(x, y, z, w) = -x^2 + y^2 \text{ is unbounded.}$$

Q. 101 Ans (1,4)

Q. 102 Ans (2)

$$f(x, y) = x^2 - y^3 \quad (1)$$

$$\frac{\partial f}{\partial y} = -3y^2 \quad (2)$$

$$\frac{\partial f}{\partial y} = 0 \Rightarrow y = 0 \text{ \& }$$

$f(x, y) = 0 \Rightarrow x^2 - 0^3 = 0 \Rightarrow x = 0$ so at point $(0, 0)$ implicit function theorem do not

work, but $f(x, y) = 0 \Rightarrow x^2 - y^3 = 0$

$$\Rightarrow y^3 = x^2 \Rightarrow y = x^{2/3}$$

Which is unique continuous function in an interval containing 0 (but not differentiable)

Q. 103 Ans (1,2,3)

Q. 104 Ans (1,3)

Q. 105 Ans (2,3)

Q. 106 Ans (1)

Note:- If x_n is n^{th} iterative value and ξ is true root of $f(x) = 0$ then if

$$\lim_{n \rightarrow \infty} \frac{|x_{n+1} - \xi|}{|x_n - \xi|^p} = \lim_{n \rightarrow \infty} \frac{\epsilon_{n+1}}{\epsilon_n^p} = \mu \text{ then order of con-}$$

vergence of iteration method is p and rate of convergence is μ .

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1}{1 + \frac{1}{n}} + \frac{1}{1 + \frac{2}{n}} + \dots + \frac{1}{1 + \frac{n}{n}} \right]$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n \frac{1}{1 + \frac{r}{n}} = \int_0^1 \frac{1}{1+x} dx$$

$$= \ln(1+x) \Big|_0^1 = \ln 2$$

$$\text{Also, } \lim_{n \rightarrow \infty} b_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim_{n \rightarrow \infty} \left| \frac{b_{n+1} - 0}{b_n - 0} \right| = \lim_{n \rightarrow \infty} \frac{\frac{1}{n+1}}{\frac{1}{n}} = 1$$

\Rightarrow Rate of convergence of $\{b_n\} = 1$ & order of convergence of $\{b_n\} = 1$.

$$\text{Also, } \left| \frac{a_{n+1} - \log 2}{a_n - \log 2} \right| = \frac{|a_n - \log 2 + a_{n+1} - a_n|}{|a_n - \log 2|}$$

$$= \frac{\left| a_n - \log 2 + \frac{1}{(2n+2)(2n+1)} \right|}{|a_n - \log 2|}$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{|a_{n+1} - \log 2|}{|a_n - \log 2|} = 1$$

\Rightarrow Rate of convergence of $\{a_n\} = 1$

& order of convergence of $\{a_n\} = 1$

Hence $\{a_n\}$ converges to $\log 2$ and has same rate of convergence of that of $\{b_n\}$

Q. 107 Ans (1,4)

If $\sigma_n = \frac{1}{n}(x_1 + x_2 + \dots + x_n)$ and if

$$x_n = 1 \text{ if } n \text{ is odd}$$

$$= 2 \text{ if } n \text{ is even}$$

then $\limsup \{x_n\} = 2$ but $\limsup(\sigma_n) = \frac{3}{2}$

So option (2) is false

but if $\{x_n\}$ is positive decreasing sequence then it is bounded below by 0 so it is convergent hence by Cauchy's first theorem on limit.

$$\lim \sigma_n = \lim_{n \rightarrow \infty} \frac{1}{n}(x_1 + x_2 + \dots + x_n) = \lim x_n$$

(option 1)

Also, in this case

$$\limsup(\sigma_n) = \liminf(\sigma_n) = \lim(\sigma_n)$$

$$= \lim(x_n) = \limsup(x_n) = \liminf(x_n)$$

By Raabe's test

$$\limsup \left\{ n \left(\frac{x_n}{x_{n+1}} - 1 \right) \right\} < 1 \text{ then all limit points}$$

of $\left\{ n \left(\frac{x_n}{x_{n+1}} - 1 \right) \right\}$ is less than 1 so $\{x_n\}$ is divergent .

(option 4)

For $x_n = 2^n$

$$\limsup \left(n \left(\frac{x_n}{x_{n+1}} - 1 \right) \right)$$

$$= \limsup \left(-\frac{n}{2} \right) = -\infty < 1 \text{ but } \{x_n\} \text{ is divergent.}$$

(option 3 is wrong)

Q. 108 Ans (2,3,4)

$$u : \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\partial_x u + 2\partial_y u = 0, \quad u(x, y) = \sin x, \quad y = 3x + 1$$

$$\frac{dx}{1} = \frac{dy}{2} = \frac{du}{0}$$

$$\Rightarrow dx = \frac{dy}{2} \Rightarrow 2x - y = C_1$$

$$dx = \frac{du}{0} \Rightarrow C_2 = u$$

$$\therefore \phi(2x - y) = u$$

$$\therefore \phi(2x - 3x - 1) = \sin x$$

$$\Rightarrow \phi(-x - 1) = \sin x \quad t = -x - 1$$

$$x = -1 - t$$

$$\Rightarrow \phi(t) = \sin(-1 - t)$$

$$\Rightarrow \sin(-2x + y - 1) = u$$

Now $V : \mathbb{R}^2 \rightarrow \mathbb{R}$

$$\partial_x V + 2\partial_y V = 0 \quad V(x, 0) = \sin x$$

$$C_1 = 2x - y \quad C_2 = V$$

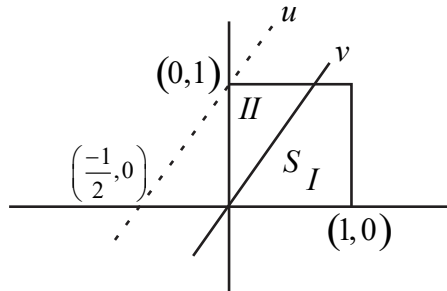
$$x = t, y = 0, V = \sin t$$

$$\therefore C_1 = 2t \quad C_2 = \sin t$$

$$C_2 = \sin\left(\frac{C_1}{2}\right)$$

$$\Rightarrow V = \sin\left(\frac{2x-y}{2}\right)$$

$$V = \sin\left(x - \frac{y}{2}\right)$$



$$\text{Now } S = [0,1] \times [0,1]$$

$$-2x + y - 1 = 0$$

$$(0,1) \& \left(-\frac{1}{2}, 0\right)$$

$$2x - y = 0 \Rightarrow (0,0), \left(\frac{1}{2}, 1\right)$$

$$(1,0) \Rightarrow V = \sin 1 > 0$$

$$(0,1) \Rightarrow V = \sin\left(-\frac{1}{2}\right) < 0$$

\therefore V changes sign in the interior of S
option (c) is correct.

$$(0,1) \Rightarrow u = \sin 0 = 0$$

$$(1,0) \Rightarrow u = \sin(-3) < 0$$

option (a) is incorrect

$$\text{If } V = 0 \Rightarrow 2x - y = 0$$

Line V lies in interior of S

option (d) is correct.

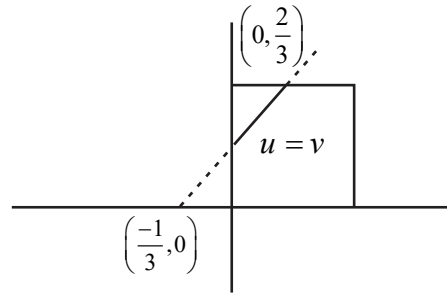
$$\frac{2x-y}{2} = -2x + y - 1$$

$$\Rightarrow 6x - 3y = -2$$

$$\therefore \left(0, \frac{2}{3}\right), \left(-\frac{1}{3}, 0\right)$$

$$\therefore u(x, y) = v(x, y)$$

along a line in S .



Q. 109 Ans (2,3)

$$(a) \quad \mathbb{Z}_2 = \{0,1\}$$

$(1+x^2)$ reducible over \mathbb{Z}_2 hence not maximal ideal.

$$(b) \quad \langle 1+x+x^2 \rangle \text{ irreducible over } \mathbb{Z}_2 \text{ so maximal ideal.}$$

$$(c) \quad \langle 1+x^2 \rangle \text{ irreducible over } \mathbb{Z}_3, \text{ so maximal ideal.}$$

$$(d) \quad \langle 1+x+x^2 \rangle \text{ reducible over } \mathbb{Z}_3 \text{ so not maximal ideal.}$$

Q. 110 Ans (2)

T_1 & T_2 are nilpotent operators

$$W_1 = \{v \in V : T_1(v) = 0\} \quad (1)$$

$$\& W_2 = \{v \in V : T_2(v) = 0\} \quad (2)$$

$$T_1 \sim T_2 \Rightarrow \text{Rank}(T_1) = \text{Rank}(T_2)$$

$$\text{Also, } \dim(W_1) = \dim V - \text{Rank}(T_1) \quad (3)$$

$$\& \dim(W_2) = \dim(V) = \text{Rank}(T_2) \quad (4)$$

$$\text{Rank}(T_1) = \text{Rank}(T_2) \Rightarrow$$

$$\dim(W_1) = \dim(W_2) \Rightarrow$$

$$W_1 \equiv W_2 \text{ (1 is correct, so do not tick it)}$$

For option 2, take

$$T_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \&$$

$$T_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\therefore \text{Rank}(T_1) = \text{Rank}(T_2) = 2$$

So $W_1 \equiv W_2$ but minimal polynomials of T_1 & T_2 are respectively $m_1(x) = x^2$ & $m_2(x) = x^3$ respectively.

So, option (2) is false hence tick it.

$T_1 = T_2 = 0 \Rightarrow T_1 \sim T_2$ so (3) is correct. do not tick it.

For option (4) if $W_1 \equiv W_2$ then

$$\therefore T_1 \text{ \& } T_2 \text{ are nilpotent operator}$$

So their characteristic polynomial are same and equal to x^n as all of their eigen values are 0.

So, option (4) is correct hence do not tick it.

Q. 111 Ans (2,3,4)

$$(a) |G| = 244 = 2^2 \times 3^2 \times 7$$

$$27 \nmid 244$$

$$\therefore G \text{ has no subgroup of order } 27$$

$$(b) |G| = 1694 = 2 \times 7 \times 11^2$$

$11^2 \mid 1694$ but $11^{2+1} \nmid 1694$ then any subgroup of order 11^2 is 11-SSG

$$\therefore \text{No of 11-SSG} = n_1 = 1 + 11k$$

$$\text{i.e. } 1 + 11k \mid 14$$

$$k = 0 \text{ then } n_{11} = 1 \text{ which is unique}$$

$$\therefore 11\text{-SSG has unique subgroup of order } 121. \\ \text{option (b) is correct .}$$

$$(c) |G| = 154 = 2 \times 7 \times 11$$

$7^1 \mid 154$ but $7^{1+1} \nmid 154$ then any subgroup of order 7 is 7-SSG

$$\text{Number of 7-SSG is } n_7 = 1 + 7k$$

$$1 + 7k \mid 22$$

$$k = 0, \text{ then } n_7 = 1$$

$$k = 3 \text{ then } n_7 = 22$$

$\therefore G$ has subgroup of order 7
but in option asked about there exist option (c)

(or $G \approx \mathbb{Z}_{154}$ thus \mathbb{Z}_{154} has unique subgroup of order 7.)

$$(d) |G| = 121$$

$$G \approx \mathbb{Z}_{121} \text{ or } G \approx \mathbb{Z}_{11} \times \mathbb{Z}_{11}$$

No. of element

$$\text{No. of subgroup of order 11 in } G = \frac{\text{of order 11}}{\phi(11)}$$

$$\therefore \text{No. of subgroup} = \frac{120}{10} = 12$$

\therefore option (d) is correct

Q. 112 Ans (1,2,3)

$$\begin{pmatrix} e & 1 & 0 \\ 0 & e \end{pmatrix}, \begin{pmatrix} e & 1 & 0 \\ 0 & e & 0 \\ 0 & 0 & e \end{pmatrix} \text{ are diagonal blocks of } A$$

so Jordan Canonical Form of A is

$$\begin{pmatrix} e & 1 & 0 & 0 & 0 \\ 0 & e & 0 & 0 & 0 \\ 0 & 0 & e & 1 & 0 \\ 0 & 0 & 0 & e & 0 \\ 0 & 0 & 0 & 0 & e \end{pmatrix}$$

So, maximum consecutive 1's on super diagonal places is 1

So, minimal polynomial of A is

$$m(x) = (x - e)^{1+1} \text{ i.e. } m(x) = (x - e)^2 \text{ \& char-}$$

acteristic polynomial of A is $C(x) = (x - e)^5$

So, A.M. of eigen values e of A is 5
(option 1)

G.M. of eigen value $e=5$ - Rank

$$(A - eI) = 5 - 2 = 3$$

(option 3)

∴ A.M. of $e \neq$ G.M. of e , so A is not diagonalisable.

Also $m(x)$ has power of $(x-e)$ more than 1 so it is not diagonalisable.
(option 2)

Q. 113 Ans (1,4)

Q. 114 Ans (2,3)

(a) $6x^2 - 13xy + 6y^2$

$$A = \begin{pmatrix} 6 & -\frac{13}{2} \\ -\frac{13}{2} & 6 \end{pmatrix}$$

$$Ch_A(x) = 6x^2 - 12x - \frac{25}{4} = 0$$

so, eigen value of A are $\frac{12 \pm \sqrt{294}}{12}$

so, number of positive terms and negative terms respectively are $p=1$ & $q=1$, hence signature = 0 & rank = 2

(b) $x^2 - xy + 2y^2$

$$B = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 2 \end{pmatrix}$$

$$Ch_B(x) = x^2 - 3x + \frac{7}{4} = 0$$

so, eigen values of B are $\frac{3 \pm \sqrt{2}}{2}$

so, number of positive terms and negative terms respectively are $p=2$ & $q=0$, hence signature = 2 & rank = 2

(c) $x^2 - xy - 2y^2$

$$C = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & -2 \end{pmatrix}$$

$$Ch_C(x) = x^2 - x - \frac{9}{4} = 0$$

so, eigen value of C are $\frac{1 \pm \sqrt{10}}{2}$

so, number of positive terms and negative terms respectively are $p=1$ & $q=1$, hence signature = 0 & rank = 2

So, $A \approx C$, B - positive definite.

Q. 115 Ans (1,2,3,4)

Q. 116 Ans (1,2)

Q. 117 Ans (2,3)

$$(P) = \begin{cases} x'(t) = \sqrt{x(t)} \\ x(0) = 0 \end{cases} \quad t > 0$$

$$(Q) = \begin{cases} y'(t) = -\sqrt{y(t)} \\ y(0) = 0 \end{cases} \quad t > 0$$

$$\frac{du}{dt} = Au^\alpha \quad y(\beta) = 0 \text{ has}$$

(i) Unique solution if $A < 0$, $\alpha \in (0,1)$ $\beta \in \mathbb{R}$

(ii) Infinite solution if $A > 0$, $\alpha \in (0,1)$ $\beta \in \mathbb{R}$
option (b) and (c) are correct

Q. 118 Ans (4)

$$g(z) = (z - z_1)(z - z_2), \dots, (z - z_n) \quad \text{and if}$$

$f(z) = 0$ is taken then

$$p(z) = \frac{1}{2\pi i} \int_C f(\xi) \times \frac{g(\xi) - g(z)}{(\xi - z)g(\xi)} d\xi = 0$$

so all option (1), (2) & (3) becomes false, hence in examination hall tick option (4) and go to the next question.

Now for option (4)

$$\begin{aligned}
 \text{As, } p(z) &= \frac{1}{2\pi i} \int_C f(\xi) \times \frac{g(\xi) - g(z)}{(\xi - z)g(\xi)} d\xi \\
 &= \frac{1}{2\pi i} \int_C \frac{f(\xi)}{\xi - z} d\xi \\
 &\quad - \frac{1}{2\pi i} \times g(z) \int_C \frac{f(\xi)}{(\xi - z)g(\xi)} d\xi \\
 \Rightarrow p(z_j) &= \frac{1}{2\pi i} \int_C \frac{f(\xi)}{\xi - z_j} d\xi \\
 &\quad - g(z_j) \times \frac{1}{2\pi i} \int_C \frac{f(\xi)}{(\xi - z_j)^2 (\xi - z_1) \dots (\xi - z_{j-1})(\xi - z_{j+1}) \dots (\xi - z_n)} d\xi \\
 &= f(z_j) \quad \quad \quad - \quad \quad \quad 0 \\
 \text{first integral} \quad \quad \quad &\quad \quad \quad \text{second integral} \\
 \text{So option (4) is correct.}
 \end{aligned}$$

Q. 119 Ans (* None are correct)

In given set of orthogonal vectors V_1, V_2, V_3 if we take V_3 as zero vector i.e. $V_3 = 0$ then For option (1) & (2) set of vectors $V_1 + V_2 + 2V_3, V_2 + V_3, V_2 + 3V_3$ will be $V_1 + V_2, V_2$ & V_2 which is dependent set of vectors so they cannot be extended to form either a basis or orthogonal basis of V because only independent set of vectors can be extended to form a basis so both option (1) & (2) are incorrect. For option (3) again set of vectors $V_1 + V_2 + 2V_3, V_2 + V_3$ & $2V_1 + V_2 + 3V_3$ will be $V_1 + V_2, V_2$ & $2V_1 + V_2$ which is linearly dependent so it cannot be extended to a basis of V because only independent set of vectors can be extended to form a basis . so option (3) is incorrect. For option (4) set of vectors $V_1 + V_2 + 2V_3, 2V_1 + V_2 + V_3$ & $2V_1 + V_2 + 3V_3$ will become $V_1 + V_2, 2V_1 + V_2$ & $2V_1 + V_2$ which is linearly dependent so it cannot be extended to a basis of V because only independent set of vectors can be extended to form a basis , so option (4) is also incorrect.

Q. 120 Ans (2,4)

If $f^{-1}: H^+ \rightarrow D$ is given by $f^{-1}(z) = \frac{z-i}{z+i}$

then it is a bijection so $f: D \rightarrow H^+$ is a bijection map hence option (b) is false.

Also if $f: D \rightarrow D$ is holomorphic function then $f(z)$ has at most one fixed point unless it is identity map.

Hence $f(\alpha) = \alpha$ & $f(\beta) = \beta$ where $\alpha \neq \beta$ implies that $f(z)$ has 2 fixed points.

So, $f(z) = z; \forall z \in D$

So statement (a) is true.

Also, $f: D \rightarrow D$ is holomorphic function &

for an $\alpha \in D$ if $f(\alpha) = \alpha$ & $f'(\alpha) = 1$ then by the concept of Schwarz lemma

$f(z) = z; \forall z \in D$

So, statement C is correct. i.e. true.
